

Andy Clark 7/17/13

# Common Core, National Assessments and *Math* *In Focus*

Meeting Common Core and World Class  
standards



# Thinking, Fast and Slow

Mathematics is an excellent medium  
for “slow thinking.”

David Kahneman

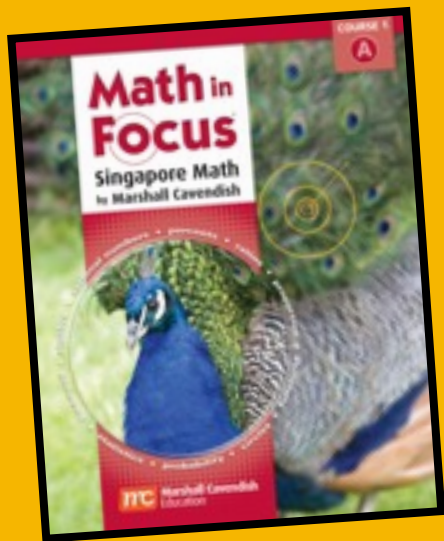
# Thinking, Fast and Slow

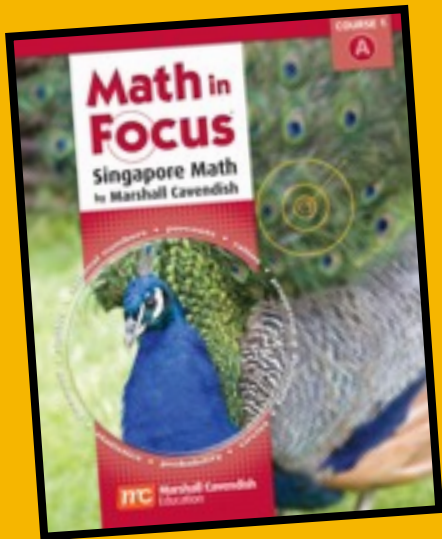
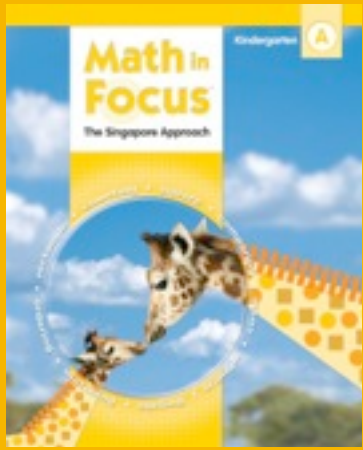
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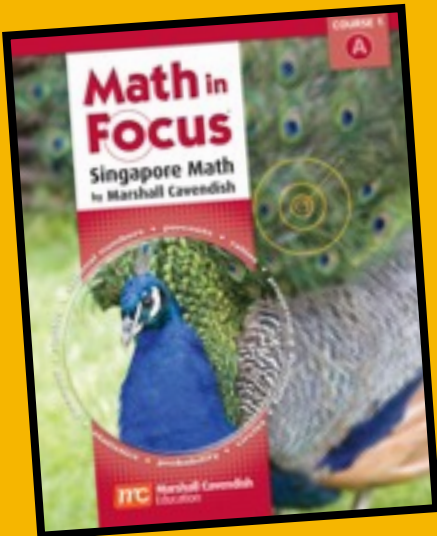
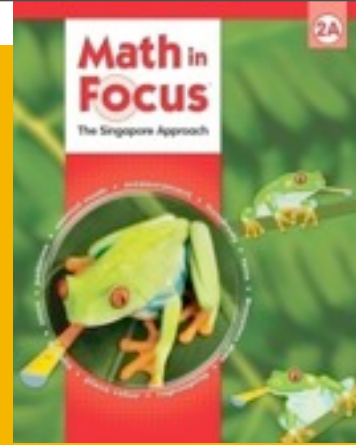
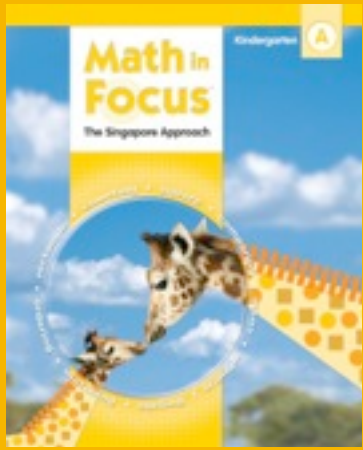
Gas hogger's old car got 12 mpg. He bought a car that now gets 14 mpg. Ecological Annie's old car got 30mpg, her new one gets 40mpg. If they drive the same amount in a year, who will save more gas?

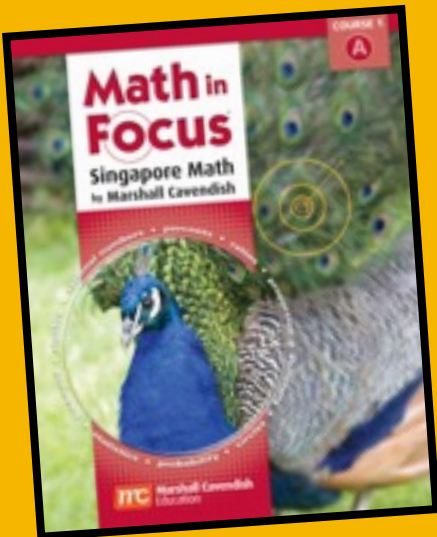
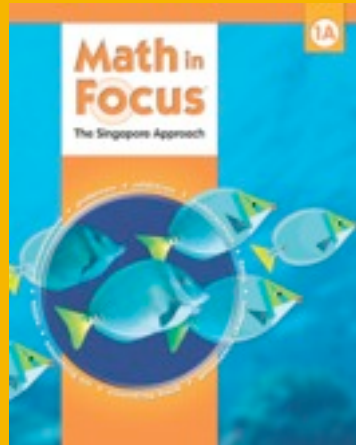
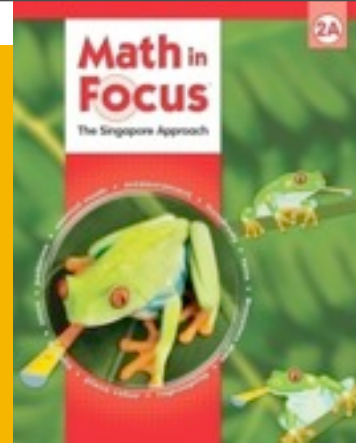
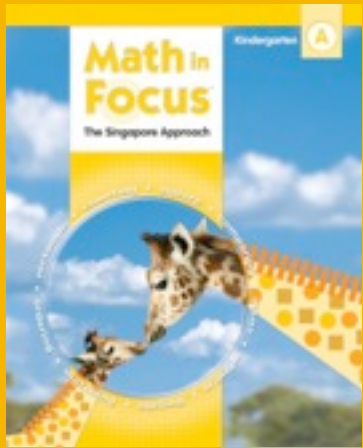
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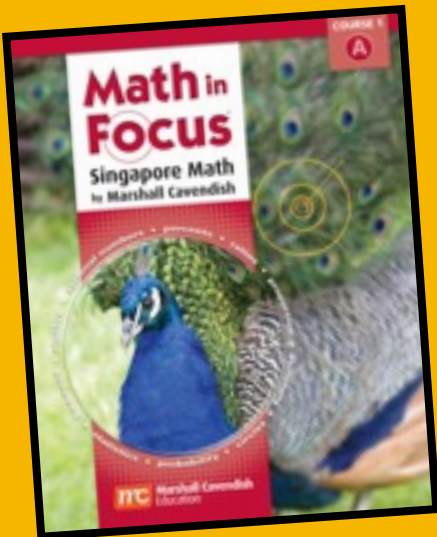
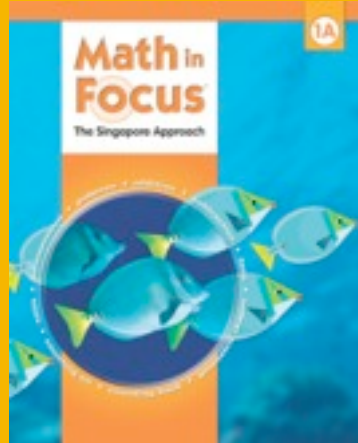
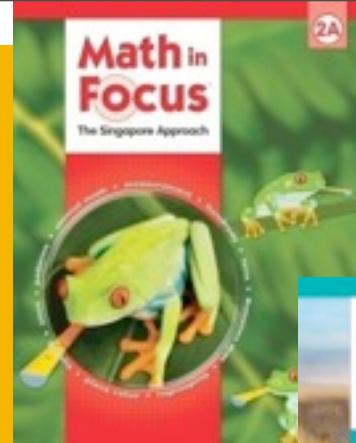
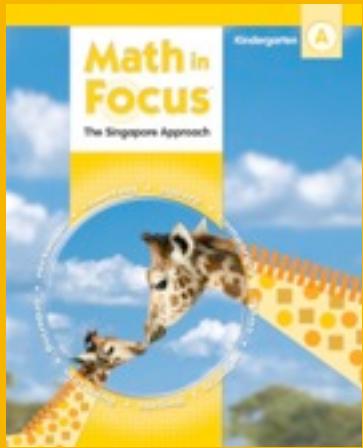




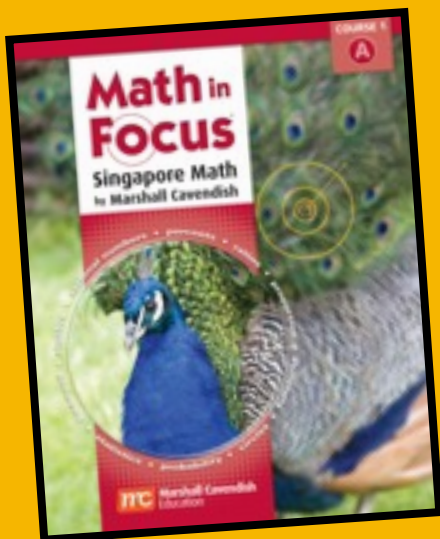
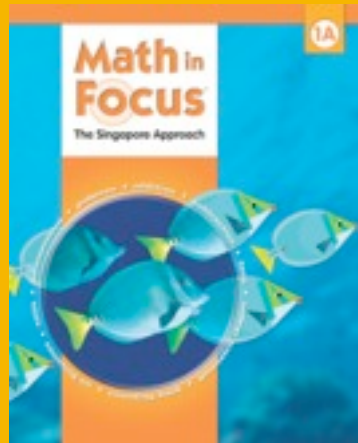
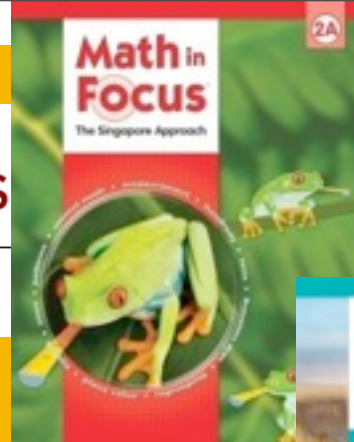
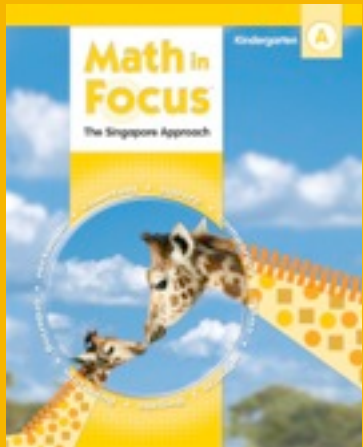




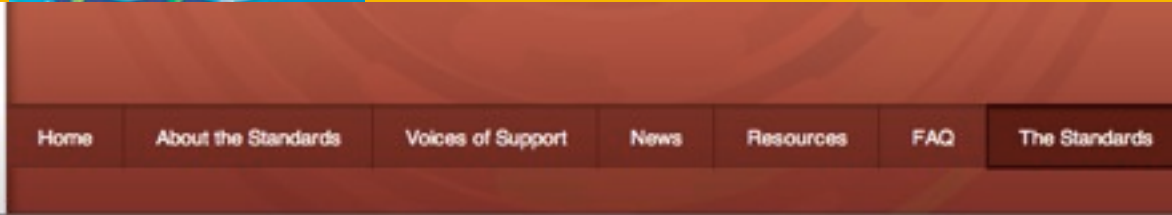
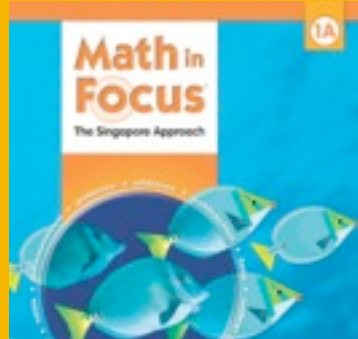
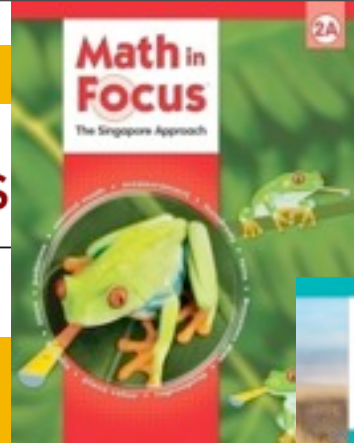
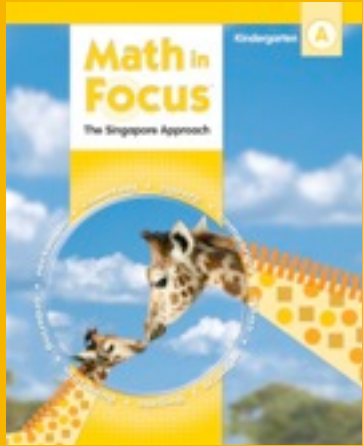




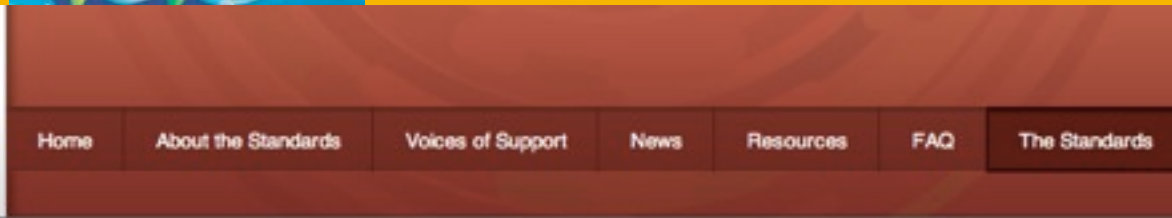
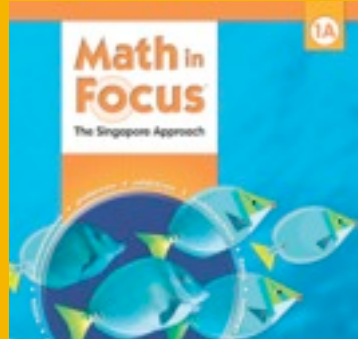
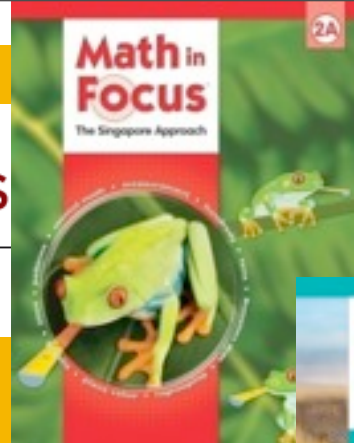
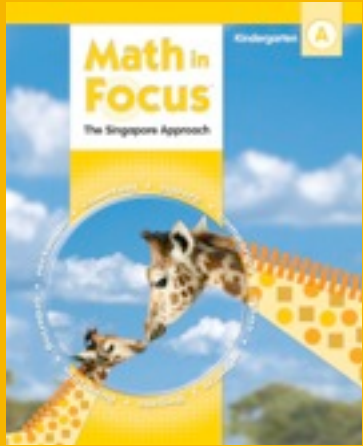
# COMMON CORE STATE STANDARDS



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## Addressing the Common Core with *Math In Focus*

# Frame for Our Work

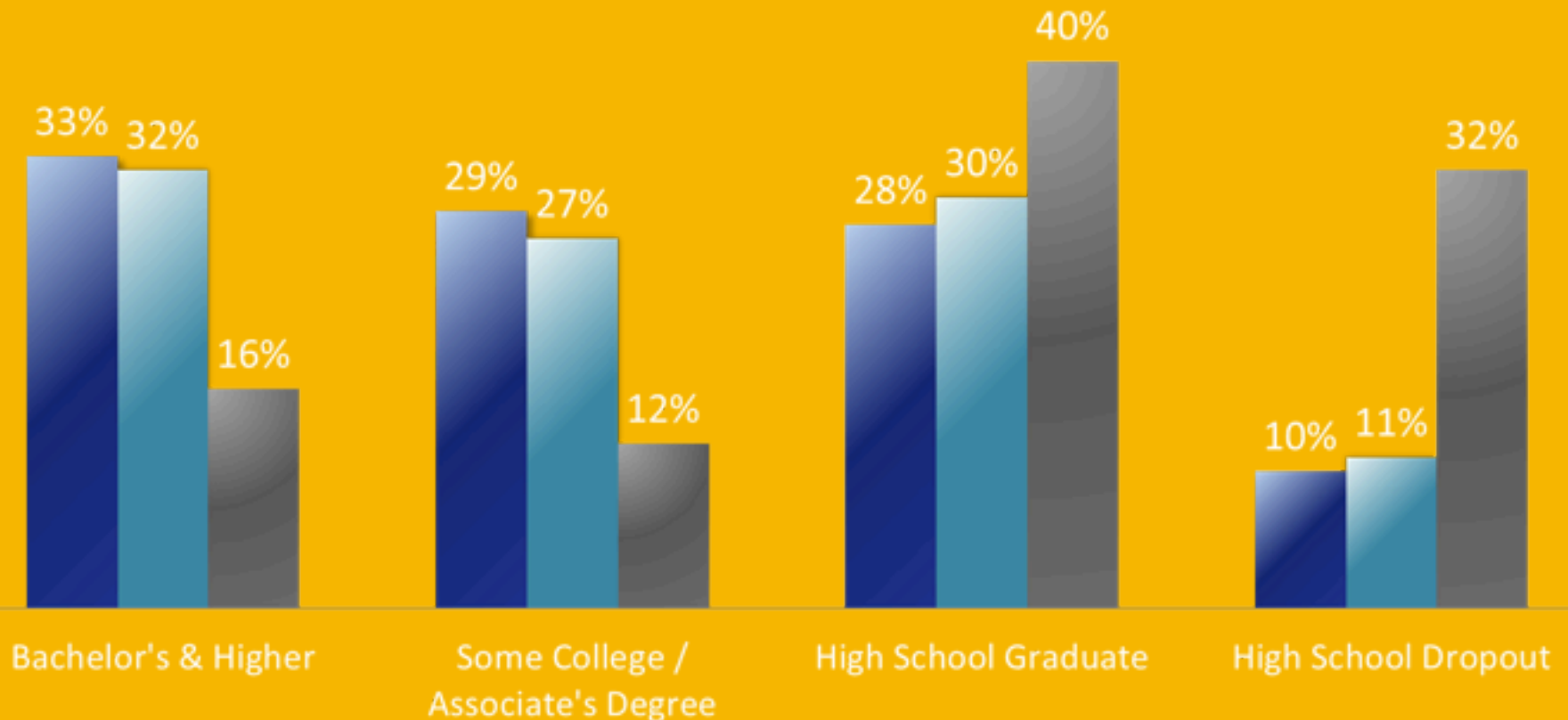
- Today's students need the math backgrounds that will enable them to compete in the world-wide economy
- The math skills required include the ability to generalize, conceptualize, apply, reason, and be creative



# Jobs in Today's (and Tomorrow's) Workforce Require More Education and Training

Percentage of Workforce by Education Level

■ 2018 ■ 2002 ■ 1973

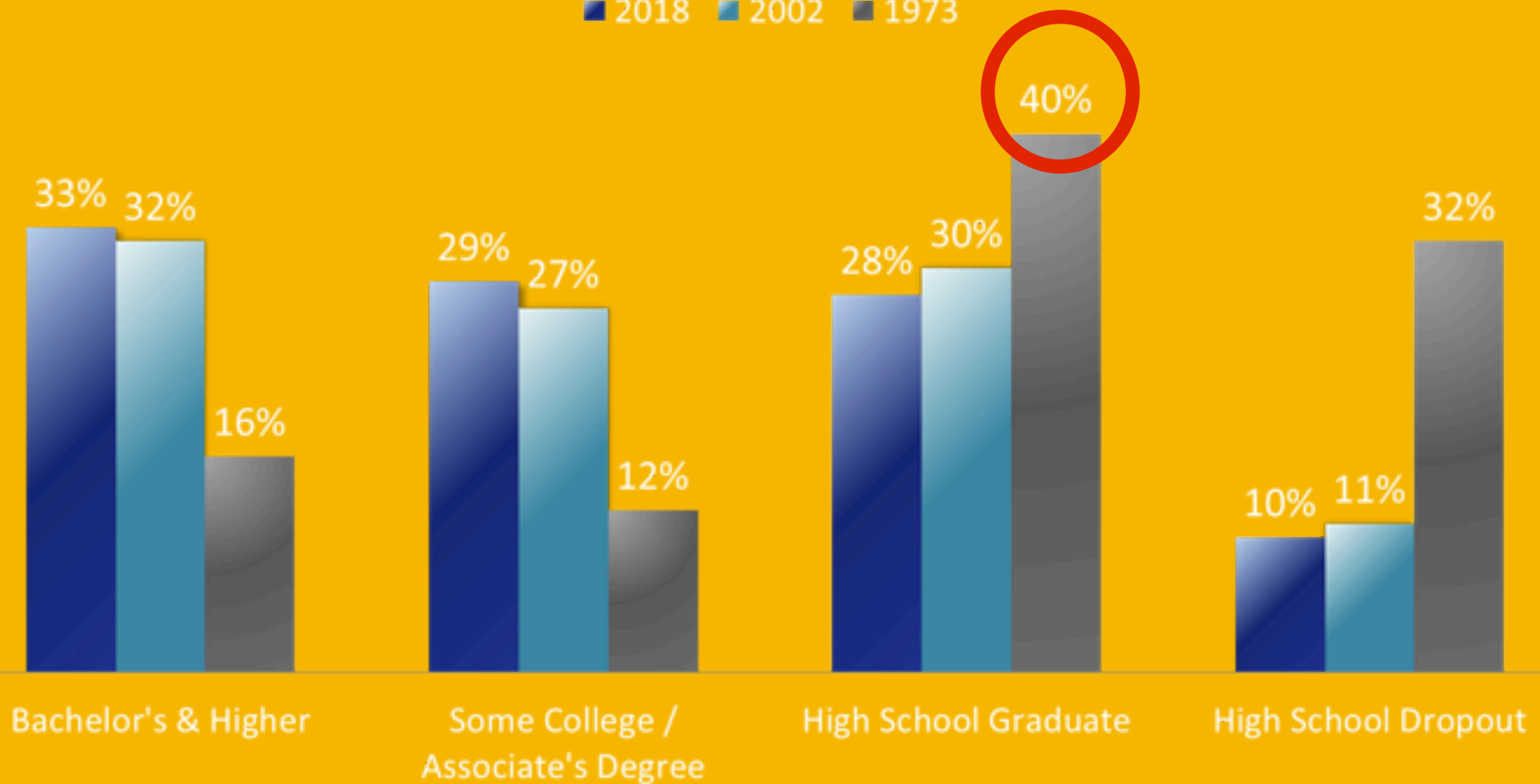


Source: Carnevale, Anthony P. et al. (June 2010). *Help Wanted: Projections of Jobs and Education Requirements Through 2018*. Georgetown Center on Education and the Workforce. [www.georgetown.edu/grad/gppi/hpi/cew/pdfs/FullReport.pdf](http://www.georgetown.edu/grad/gppi/hpi/cew/pdfs/FullReport.pdf)

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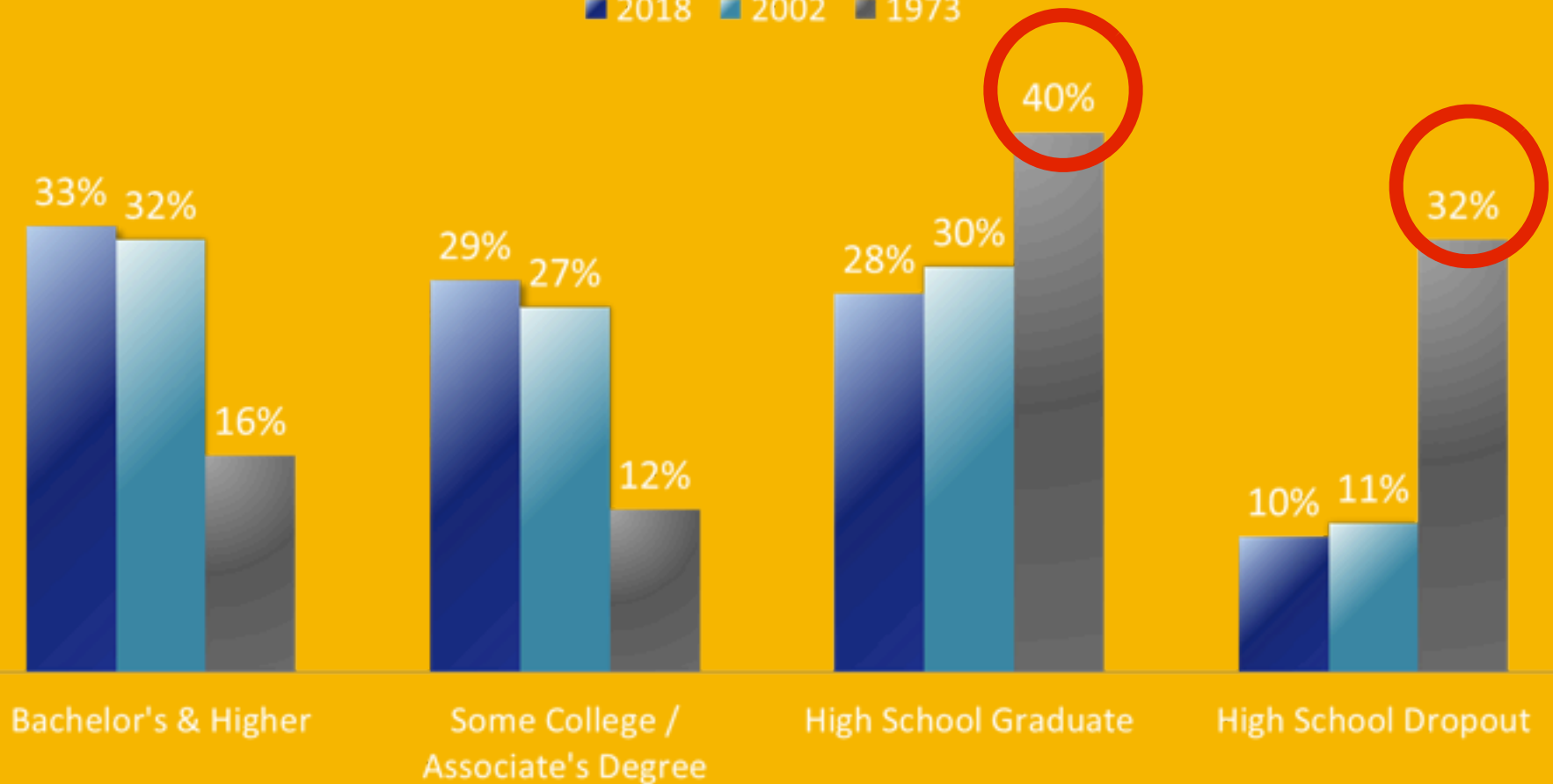


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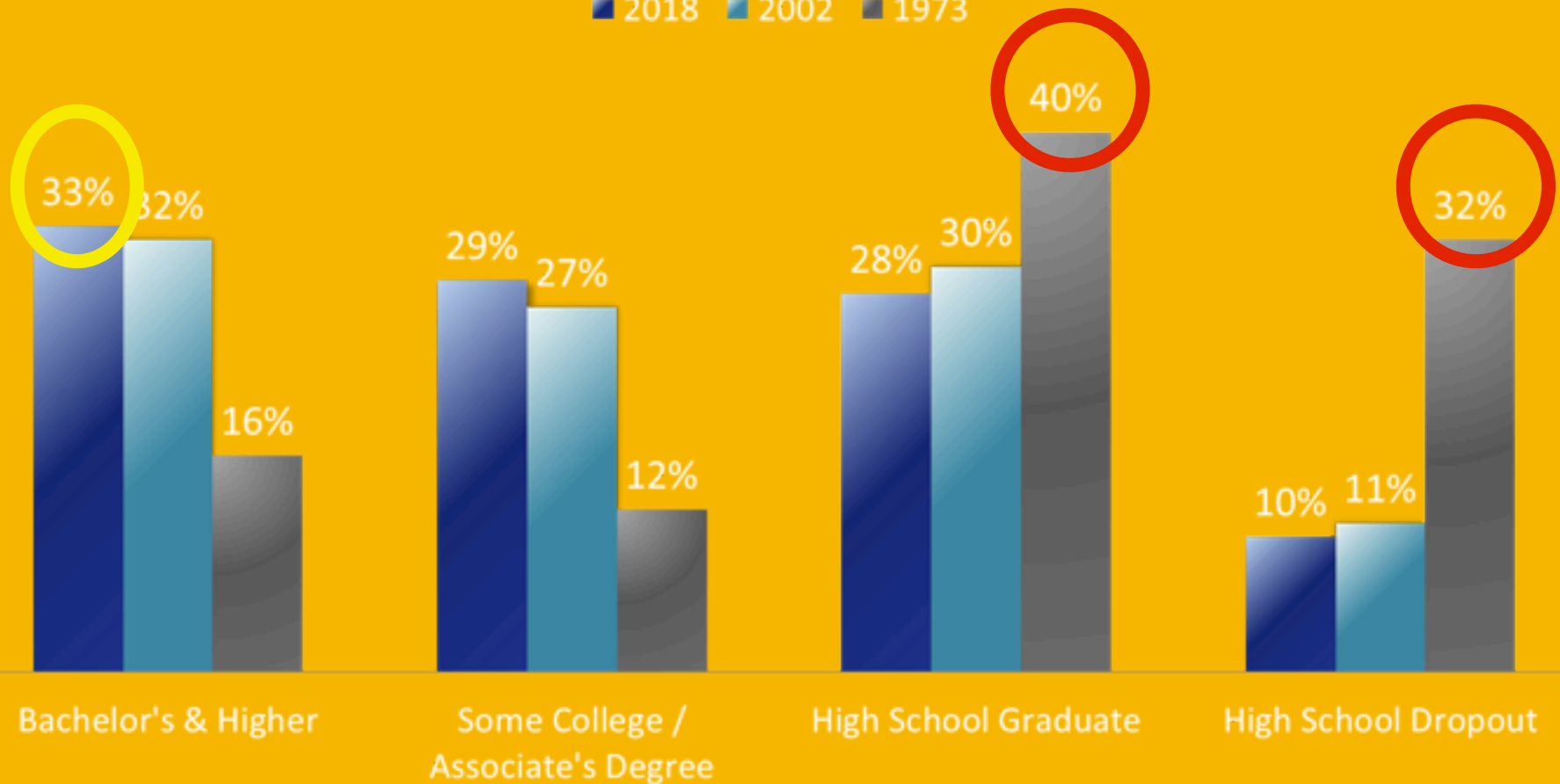
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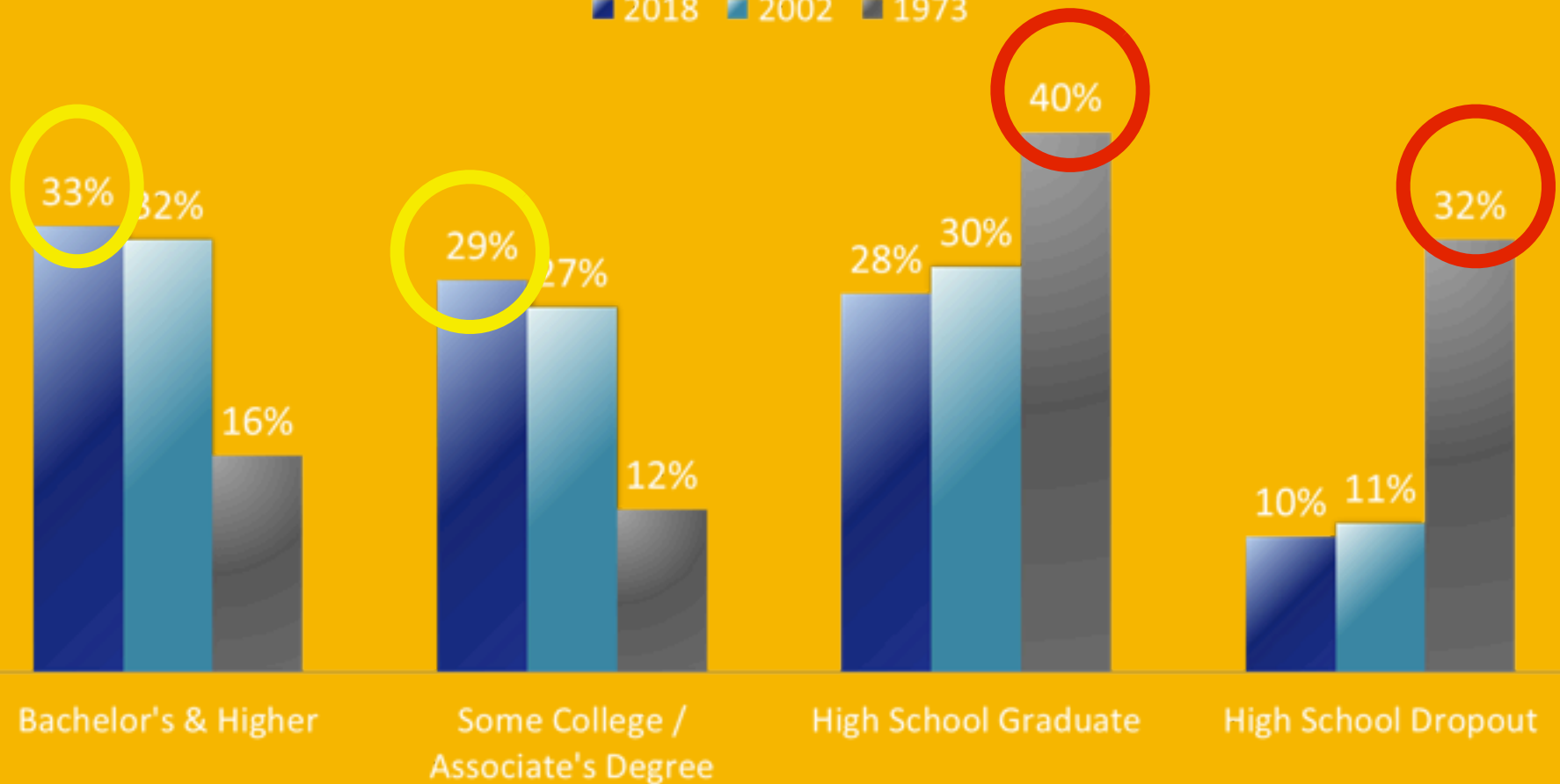


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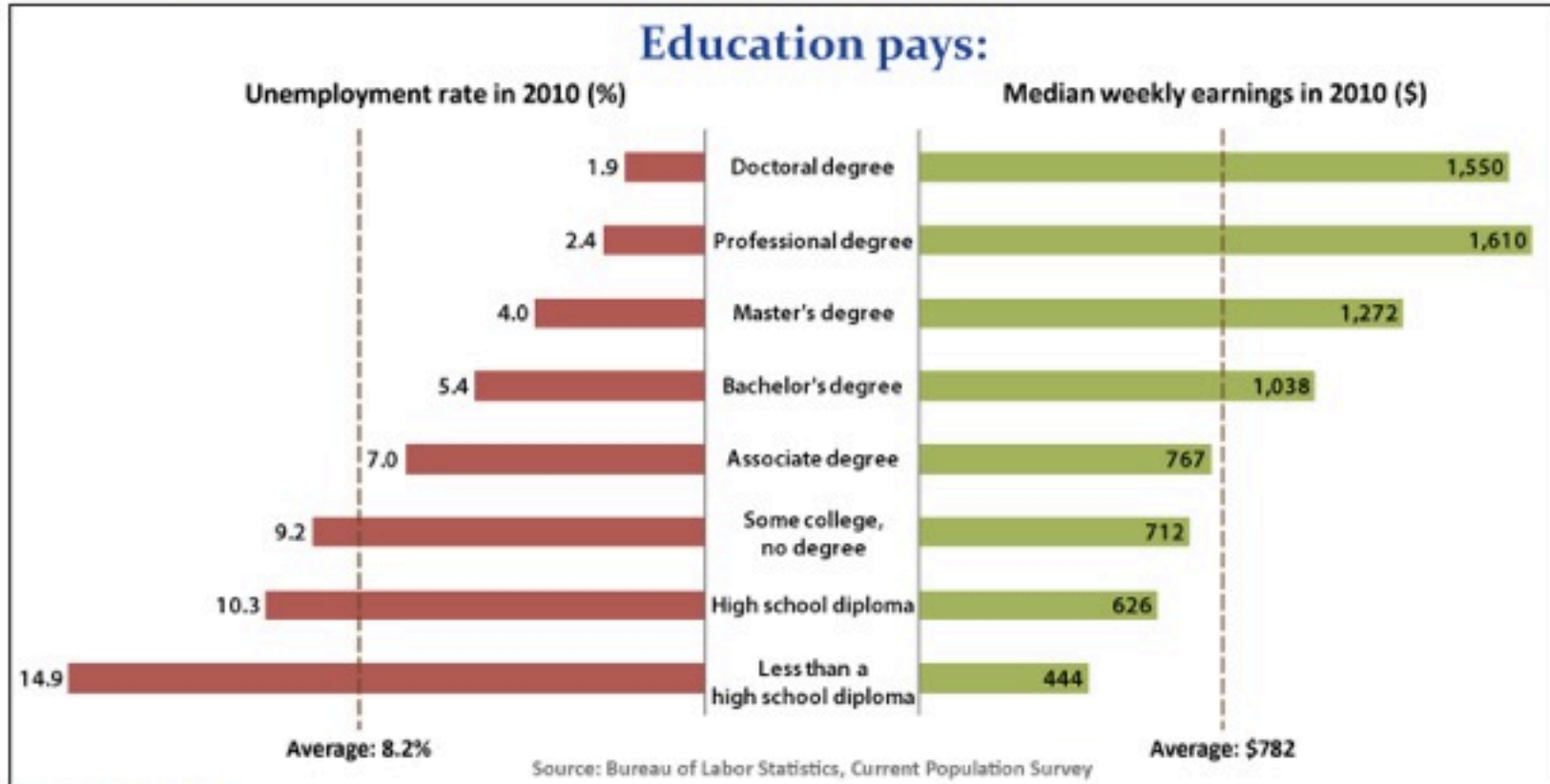
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# Education pays ...

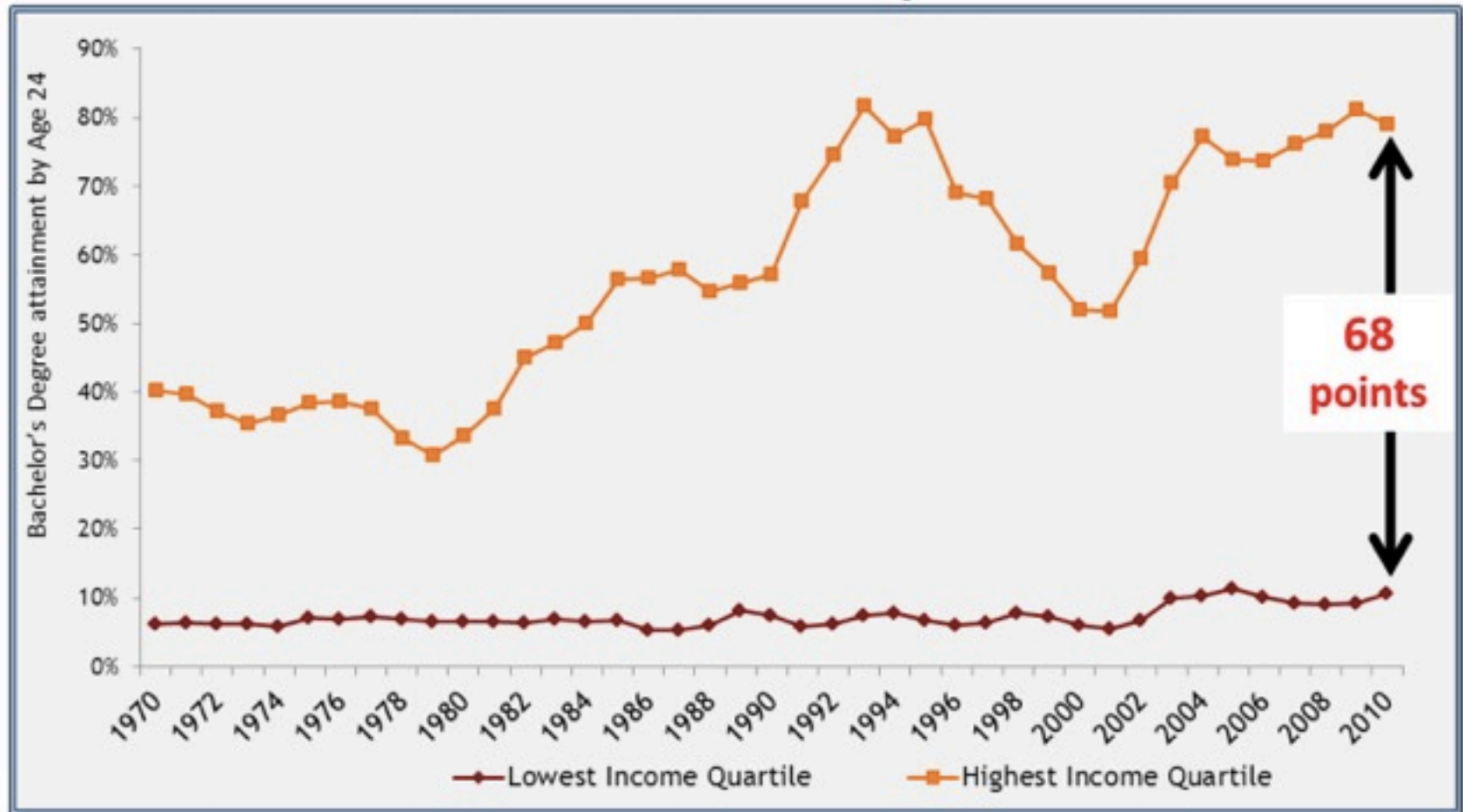
Education pays in higher earnings and lower unemployment rates



[\[Chart data—TXT\]](#)

Note: Data are 2010 annual averages for persons age 25 and over. Earnings are for full-time wage and salary workers.

Bachelor's attainment for high-income young people is **68 points** higher than for low-income youth.



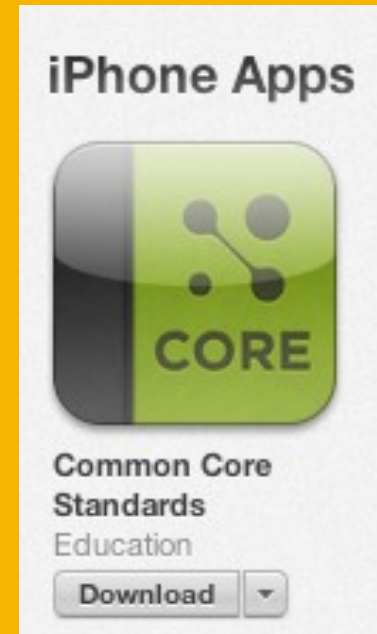
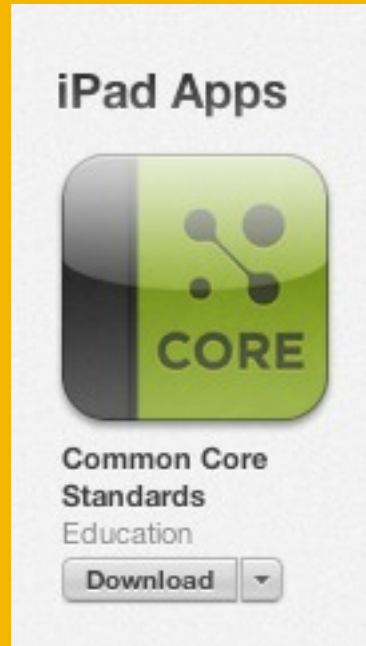
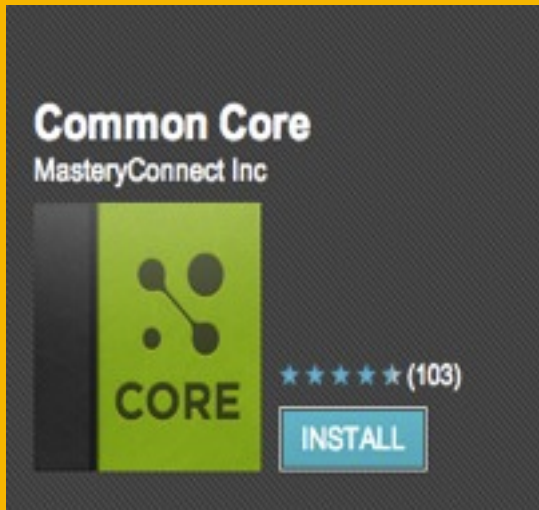
Source: "Bachelor's Degree Attainment by Age 24 by Family Income Quartiles, 1970 to 2010."  
Graphic: The Education Trust

# Frame for Our Work

- Today's students need the math backgrounds that will enable them to compete in the world-wide economy
- The math skills required include the ability to generalize, conceptualize, apply, reason, and be creative
- Therefore must be made accessible to all



# Download the standards



# Common Core key points



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- Solid foundation in number in K-5



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- Stresses procedural skill and conceptual understanding and expects mastery of concepts each year
- Strong foundation at grades K-5 allow for more hands on activities in algebra, geometry and statistics in 6 - 8
- Students are well prepared for algebra in either 8th or 9th grade

# Common Core Instructional emphases

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- Focus: Prioritize topics and focus on them
- Coherence: Topics taught to mastery so each grade level builds on the previous ones
- Depth of understanding: Concepts are taught so students understand how, why, and when they work and how to apply them
- Balance conceptual understanding and procedural fluency: students understand concepts deeply and become fluent with procedures and facts

# What must an effective curriculum do then?

**COMMON CORE  
STATE STANDARDS** FOR

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**Mathematics**

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According to Common Core has the following features:

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## **Answer Getting vs Learning Mathematics**

**COMMON CORE  
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**Mathematics**

“The Standards for Mathematical Content are a balanced combination of procedure and understanding.”



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**STATE STANDARDS** FOR

**Mathematics**

# Balance understanding and procedural fluency

“Expectations that begin with the word “understand” are often especially good opportunities to connect the practices to the content. Students who lack understanding of a topic may rely on procedures too heavily.”

COMMON CORE  
STATE STANDARDS FOR

Mathematics



# Balance understanding and procedural fluency

Use place value understanding and properties of operations to add and subtract.

5. Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

7. Add and subtract within 1000 using concrete models or drawing and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction

# Balance understanding and procedural fluency

$$\begin{array}{r} 182 \\ +93 \\ \hline \end{array}$$

# Balance understanding and procedural fluency






**Math in Focus**

Singapore Math by Marshall Cavendish

# Balance understanding and procedural fluency

**Learn** You can add using base-ten blocks and a place-value chart to regroup tens.

$182 + 93 = ?$

	Hundreds	Tens	Ones
182			
93			

**Step 1**  
Add the ones.




$$\begin{array}{r} 182 \\ + 93 \\ \hline 5 \end{array}$$

2 ones + 3 ones = 5 ones

**Step 2**  
Add the tens.

$$\begin{array}{r} 182 \\ + 93 \\ \hline 75 \end{array}$$

8 tens + 9 tens = 17 tens  
Regroup the tens.  
17 tens = 1 hundred 7 tens

	Hundreds	Tens	Ones
275			

**Step 3**  
Add the hundreds.

$$\begin{array}{r} 182 \\ + 93 \\ \hline 275 \end{array}$$

1 hundred + 1 hundred = 2 hundreds

So,  $182 + 93 = 275$ .

**Math in Focus**

Singapore Math by Marshall Cavendish

# Balance understanding and procedural fluency

**Math in Focus**

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# Balance understanding and procedural fluency

Learn

**Model division with regrouping in hundreds, tens, and ones.**

A farmer sells his crops to 3 restaurants. He divides 525 heads of lettuce equally among the 3 restaurants. How many heads of lettuce does each restaurant receive?



# Use an effective instructional strategy: Concrete - Pictorial - Abstract

$$3 \overline{) 525}$$

# Use an effective instructional strategy: Concrete - Pictorial - Abstract

$$\begin{array}{r} 1 \\ \hline 3 \overline{) 525} \end{array}$$



# Use an effective instructional strategy: Concrete - Pictorial - Abstract

$$\begin{array}{r} 1 \\ \hline 3 \overline{) 525} \\ \underline{300} \end{array}$$

# Use an effective instructional strategy: Concrete - Pictorial - Abstract

$$\begin{array}{r} 1 \\ \hline 3 \overline{) 525} \\ \underline{300} \\ 225 \end{array}$$

# Balance understanding and procedural fluency

## Lesson 3.3 Modeling Division with Regrouping

### Lesson Objectives

- Model regrouping in division.
- Divide a 3-digit number by a 1-digit number with regrouping.

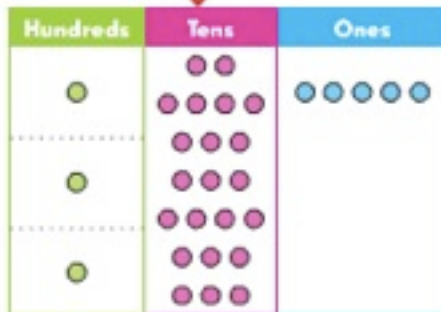
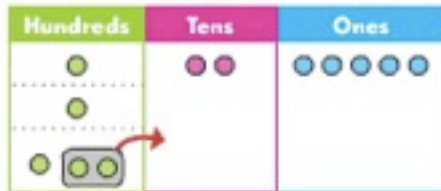
**Vocabulary**  
regroup

### Learn Model division with regrouping in hundreds, tens, and ones.

A farmer sells his crops to 3 restaurants. He divides 525 heads of lettuce equally among the 3 restaurants. How many heads of lettuce does each restaurant receive?



$$525 \div 3 = ?$$



### Step 1

Divide the hundreds by 3.

5 hundreds  $\div$  3 = 1 hundred with 2 hundreds left over

$$\begin{array}{r} 1 \\ 3 \overline{) 525} \\ \underline{300} \\ 2 \end{array}$$

**Regroup** the hundreds.

2 hundreds = 20 tens

Add the tens.

20 tens + 2 tens = 22 tens

$$\begin{array}{r} 1 \\ 3 \overline{) 525} \\ \underline{300} \\ 225 \end{array}$$

# Balance understanding and procedural fluency

**Step 1**  
Divide the hundreds by 3.  
 $5 \text{ hundreds} \div 3 = 1 \text{ hundred}$  with 2 hundreds left over.

**Step 2**  
Divide the tens by 3.  
 $22 \text{ tens} \div 3 = 7 \text{ tens}$  with 1 ten left over.

Regroup the ten.  
 $1 \text{ ten} = 10 \text{ ones}$

Add the ones.  
 $10 \text{ ones} + 5 \text{ ones} = 15 \text{ ones}$

**Step 3**  
Divide the ones by 3.  
 $15 \text{ ones} \div 3 = 5 \text{ ones}$

So,  $525 \div 3 = 175$ .  
Each restaurant receives 175 heads of lettuce.

**Long Division Problems:**

$$\begin{array}{r} 175 \\ 3 \overline{) 525} \\ \underline{300} \phantom{0} \\ 225 \phantom{0} \\ \underline{210} \phantom{0} \\ 150 \\ \underline{150} \\ 0 \end{array}$$

$$\begin{array}{r} 175 \\ 3 \overline{) 525} \\ \underline{300} \phantom{0} \\ 225 \phantom{0} \\ \underline{210} \phantom{0} \\ 150 \\ \underline{150} \\ 0 \end{array}$$

# Balance understanding and procedural fluency

But what does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, *why* a particular mathematical statement is true or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as  $(a + b)(x + y)$  and a student who can explain where the mnemonic comes from... Understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness.

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## Answer Getting vs Learning Mathematics





1  $732 - 218 = 516$

Is the answer correct?

Show how you would check it.

**Tell how you could solve this problem.  
Then solve it.**

**How could you check your answer?**

2 There are 781 rooms in a hotel.

472 rooms are white.

The rest are blue.

How many rooms are blue?





1

$$732 - 218 = 516$$

How do you know?

Is the answer correct?

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There are 781 rooms in a hotel.

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1  $732 - 218 = 516$

How do you know?

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Show how you would check it.

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**Is it reasonable?**

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2 There are 781 rooms in a hotel.

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# Standards emphasize big ideas



“These Standards endeavor to follow such a design, not only by stressing conceptual understanding of key ideas, but also by continually returning to organizing principles such as place value or the properties of operations to structure those ideas.”



Find the correct place-value chart for the number.


3

15



Tens	Ones
	

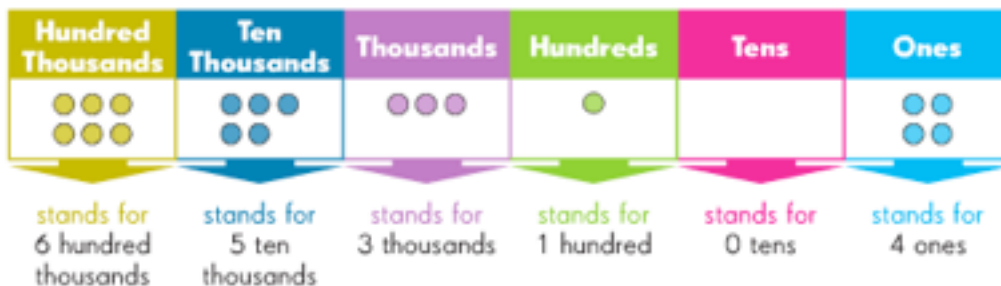
Tens	Ones
	

Tens	Ones
	

Learn

## Write numbers in **standard form** and **word form**.

What is the number in standard form and word form?



	Standard Form	Word Form
6 hundred thousands	600,000	six hundred thousand
5 ten thousands	50,000	fifty thousand
3 thousands	3,000	three thousand
1 hundred	100	one hundred
0 tens	0	
4 ones	4	four

Number in standard form: 653,104

Number in word form: six hundred fifty-three thousand, one hundred four





## Multiply decimals by 10.

Look at these examples.

	Hundreds	Tens	Ones	Tenths	Hundredths
12		●	●●		
$12 \times 10$	●	●●			
1			●		
$1 \times 10$		●			
0.1				●	
$0.1 \times 10$			●		
0.12				●	●●
$0.12 \times 10$			●	●●	



	Hundreds	Tens	Ones	Tenths	Hundredths
12		1	2		
$12 \times 10$	1	2	0		
1			1		
$1 \times 10$		1	0		
0.1			0	1	
$0.1 \times 10$			1	0	
0.12			0	1	2
$0.12 \times 10$			1	2	0

What happens to the digits of the decimal when it is multiplied by 10?  
Each digit moves 1 place to the left in the place-value chart.



# Standards are prioritized

Areas deserving intense focus are listed below:

Grade	Priorities in Support of Rich Instruction and Expectations of Fluency and Conceptual Understanding
K–2	Addition and subtraction, measurement using whole number quantities
3–5	Multiplication and division of whole numbers and fractions
6	Ratios and proportional reasoning; early expressions and equations
7	Ratios and proportional reasoning; arithmetic of rational numbers
8	Linear algebra <span style="float: right;">Align Text</span>



# Standards examples

## Operations and Algebraic Thinking

3.OA

### Represent and solve problems involving multiplication and division.

1. Interpret products of whole numbers, e.g., interpret  $5 \times 7$  as the total number of objects in 5 groups of 7 objects each. *For example, describe a context in which a total number of objects can be expressed as  $5 \times 7$ .*
2. Interpret whole-number quotients of whole numbers, e.g., interpret  $56 \div 8$  as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each. *For example, describe a context in which a number of shares or a number of groups can be expressed as  $56 \div 8$ .*
3. Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays, and measurement quantities, e.g., by using drawings and equations with a symbol for the unknown number to represent the problem.<sup>1</sup>
4. Determine the unknown whole number in a multiplication or division equation relating three whole numbers. *For example, determine the unknown number that makes the equation true in each of the equations  $8 \times ? = 48$ ,  $5 = \square \div 3$ ,  $6 \times 6 = ?$ .*

# Standards examples

## Number and Operations—Fractions

5.NF

4. Apply and extend previous understandings of multiplication to multiply a fraction or whole number by a fraction.
  - a. Interpret the product  $(a/b) \times q$  as  $a$  parts of a partition of  $q$  into  $b$  equal parts; equivalently, as the result of a sequence of operations  $a \times q \div b$ . *For example, use a visual fraction model to show  $(2/3) \times 4 = 8/3$ , and create a story context for this equation. Do the same with  $(2/3) \times (4/5) = 8/15$ . (In general,  $(a/b) \times (c/d) = ac/bd$ .)*
5. Interpret multiplication as scaling (resizing), by:
  - a. Comparing the size of a product to the size of one factor on the basis of the size of the other factor, without performing the indicated multiplication.
  - b. Explaining why multiplying a given number by a fraction greater than 1 results in a product greater than the given number (recognizing multiplication by whole numbers greater than 1 as a familiar case); explaining why multiplying a given number by a fraction less than 1 results in a product smaller than the given number; and relating the principle of fraction equivalence  $a/b = (n \times a)/(n \times b)$  to the effect of multiplying  $a/b$  by 1.

# Lesson 4.1 Multiplying Proper Fractions

## Lesson Objective

- Multiply proper fractions.

### Vocabulary

product

common factor

## Learn Use models to multiply fractions.

Find  $\frac{1}{2} \times \frac{2}{3}$ .

Margie drew a rectangle and colored  $\frac{2}{3}$  of it blue.

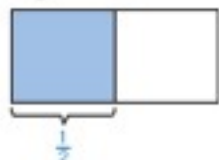


She then drew stripes over  $\frac{1}{2}$  of the colored parts.

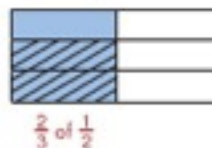


$$\begin{aligned} \frac{1}{2} \times \frac{2}{3} &= \frac{1}{2} \text{ of } \frac{2}{3} \\ &= \frac{1}{3} \leftarrow \text{Number of parts with stripes} \\ &= \frac{1}{3} \leftarrow \text{Total number of parts} \end{aligned}$$

Paul drew an identical rectangle and colored  $\frac{1}{2}$  of it blue.



He then drew stripes over  $\frac{2}{3}$  of the colored part.



$$\begin{aligned} \frac{2}{3} \times \frac{1}{2} &= \frac{2}{3} \text{ of } \frac{1}{2} \\ &= \frac{1}{3} \leftarrow \text{Number of parts with stripes} \\ &= \frac{1}{3} \leftarrow \text{Total number of parts} \end{aligned}$$

Margie and Paul get the same **product**:  $\frac{1}{3}$ .

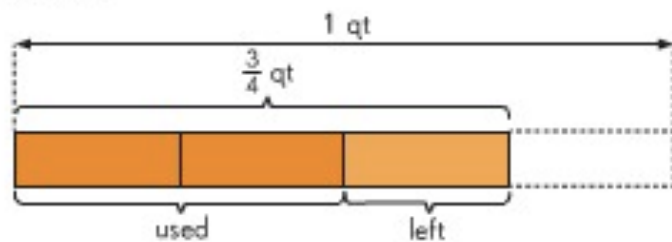
So,  $\frac{1}{2} \times \frac{2}{3} = \frac{2}{3} \times \frac{1}{2}$ .

Learn

**Multiply fractions to solve real-world problems.**

Maurice has  $\frac{3}{4}$  quart of chicken stock. He uses  $\frac{2}{3}$  of it to make some soup.

- How much chicken stock does he use to make the soup?
- How much chicken stock does he have left?

**Method 1**

The model shows that:

4 units  $\rightarrow$  1 qt

1 unit  $\rightarrow$   $\frac{1}{4}$  qt

2 units  $\rightarrow$   $\frac{1}{2}$  qt

- Maurice uses  $\frac{1}{2}$  quart of chicken stock to make the soup.
- He has  $\frac{1}{4}$  quart of chicken stock left.

**Method 2**

$$\text{a } \frac{2}{3} \times \frac{3}{4} = \frac{6}{12} = \frac{1}{2}$$

Maurice uses  $\frac{1}{2}$  quart of chicken stock to make the soup.

$$\begin{aligned} \text{b } \frac{3}{4} - \frac{1}{2} &= \frac{3}{4} - \frac{2}{4} \\ &= \frac{1}{4} \end{aligned}$$

He has  $\frac{1}{4}$  quart of chicken stock left.

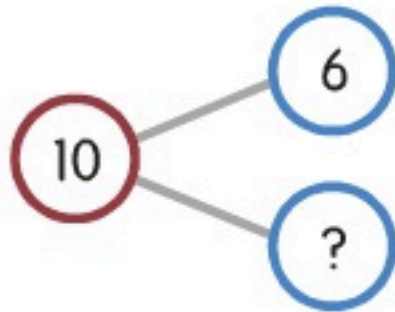
TABLE 1. Common addition and subtraction situations.<sup>4</sup>

	Result Unknown	Change Unknown	Start Unknown
<b>Add to</b>	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = ?$	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over to the first two? $2 + ? = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before? $? + 3 = 5$
<b>Take from</b>	Five apples were on the table. I ate two apples. How many apples are on the table now? $5 - 2 = ?$	Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat? $5 - ? = 3$	Some apples were on the table. I ate two apples. Then there were three apples. How many apples were on the table before? $? - 2 = 3$
	Total Unknown	Addend Unknown	Both Addends Unknown <sup>1</sup>
<b>Put Together/ Take Apart<sup>2</sup></b>	Three red apples and two green apples are on the table. How many apples are on the table? $3 + 2 = ?$	Five apples are on the table. Three are red and the rest are green. How many apples are green? $3 + ? = 5$ , $5 - 3 = ?$	Grandma has five flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5$ , $5 = 5 + 0$ $5 = 1 + 4$ , $5 = 4 + 1$ $5 = 2 + 3$ , $5 = 3 + 2$
	Difference Unknown	Bigger Unknown	Smaller Unknown
<b>Compare<sup>3</sup></b>	(“How many more?” version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy?  (“How many fewer?” version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5$ , $5 - 2 = ?$	(Version with “more”): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have?  (Version with “fewer”): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?$ , $3 + 2 = ?$	(Version with “more”): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have?  (Version with “fewer”): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?$ , $? + 3 = 5$

<sup>1</sup>These take apart situations can be used to show all the decompositions of a given number. To have the total on the left of the equal sign, help children understand that the  $=$  sign does not

## Subtract to solve word problems by taking away.

There are 10 biscuits on a plate.  
Luis takes some.  
6 biscuits are left.  
How many biscuits does he take?



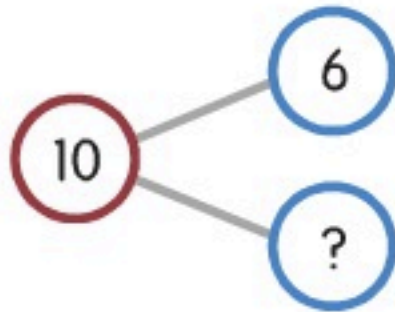
$$10 - 6 = 4$$

Luis takes 4 biscuits.



**Subtract to solve word problems by taking away.**

There are 10 biscuits on a plate.  
Luis takes some.  
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How many biscuits does he take?



$$10 - 6 = 4$$

Luis takes 4 biscuits.

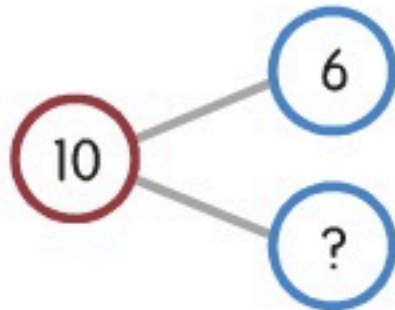


Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat?

$$5 - ? = 3$$

**Subtract to solve word problems by taking away.**

There are 10 biscuits on a plate.  
Luis takes some.  
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$$10 - 6 = 4$$

Luis takes 4 biscuits.



Five apples were on the table. I ate some apples. Then there were three apples. How many apples did I eat?

$$5 - ? = 3$$

## Change Unknown



459 children were at the library yesterday.  
46 fewer children are at the library today.  
How many children are at the library today?

459 children were at the library yesterday.  
 46 fewer children are at the library today.  
 How many children are at the library today?

	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare <sup>1</sup>	(“How many more?” version): Lucy has two apples. Julie has five apples. How many more apples does Julie have than Lucy?	(Version with “more”): Julie has three more apples than Lucy. Lucy has two apples. How many apples does Julie have?	(Version with “more”): Julie has three more apples than Lucy. Julie has five apples. How many apples does Lucy have?
	(“How many fewer?” version): Lucy has two apples. Julie has five apples. How many fewer apples does Lucy have than Julie? $2 + ? = 5$ , $5 - 2 = ?$	(Version with “fewer”): Lucy has 3 fewer apples than Julie. Lucy has two apples. How many apples does Julie have? $2 + 3 = ?$ , $3 + 2 = ?$	(Version with “fewer”): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have? $5 - 3 = ?$ , $? + 3 = 5$

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## Smaller Unknown

TABLE 2. Common multiplication and division situations.<sup>7</sup>

	Unknown Product	Group Size Unknown ("How many in each group?" Division)	Number of Groups Unknown ("How many groups?" Division)
	$3 \times 6 = ?$	$3 \times ? = 18$ , and $18 \div 3 = ?$	$? \times 6 = 18$ , and $18 \div 6 = ?$
Equal Groups	<p>There are 3 bags with 6 plums in each bag. How many plums are there in all?</p> <p><i>Measurement example.</i> You need 3 lengths of string, each 6 inches long. How much string will you need altogether?</p>	<p>If 18 plums are shared equally into 3 bags, then how many plums will be in each bag?</p> <p><i>Measurement example.</i> You have 18 inches of string, which you will cut into 3 equal pieces. How long will each piece of string be?</p>	<p>If 18 plums are to be packed 6 to a bag, then how many bags are needed?</p> <p><i>Measurement example.</i> You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?</p>
Arrays, <sup>4</sup> Area <sup>5</sup>	<p>There are 3 rows of apples with 6 apples in each row. How many apples are there?</p> <p><i>Area example.</i> What is the area of a 3 cm by 6 cm rectangle?</p>	<p>If 18 apples are arranged into 3 equal rows, how many apples will be in each row?</p> <p><i>Area example.</i> A rectangle has area 18 square centimeters. If one side is 3 cm long, how long is a side next to it?</p>	<p>If 18 apples are arranged into equal rows of 6 apples, how many rows will there be?</p> <p><i>Area example.</i> A rectangle has area 18 square centimeters. If one side is 6 cm long, how long is a side next to it?</p>
Compare	<p>A blue hat costs \$6. A red hat costs 3 times as much as the blue hat. How much does the red hat cost?</p> <p><i>Measurement example.</i> A rubber band is 6 cm long. How long will the rubber band be when it is stretched to be 3 times as long?</p>	<p>A red hat costs \$18 and that is 3 times as much as a blue hat costs. How much does a blue hat cost?</p> <p><i>Measurement example.</i> A rubber band is stretched to be 18 cm long and that is 3 times as long as it was at first. How long was the rubber band at first?</p>	<p>A red hat costs \$18 and a blue hat costs \$6. How many times as much does the red hat cost as the blue hat?</p> <p><i>Measurement example.</i> A rubber band was 6 cm long at first. Now it is stretched to be 18 cm long. How many times as long is the rubber band now as it was at first?</p>
General	$a \times b = ?$	$a \times ? = p$ , and $p \div a = ?$	$? \times b = p$ , and $p \div b = ?$

<sup>4</sup>The language in the array examples shows the easiest form of array problems. A harder form is to use the terms rows and columns: The apples in the grocery window are in 3 rows and 6 columns. How many apples are in there? Both forms are valuable.

<sup>5</sup>Area involves arrays of squares that have been pushed together so that there are no gaps or overlaps, so array problems include these especially important measurement situations.

Benny has to store 36 stuffed animals in boxes.  
He puts 9 stuffed animals in each box.

How many boxes does Benny use?

$$36 \div 9 = 4$$

Benny uses 4 boxes.

**Math in Focus**

Singapore Math by Marshall Cavendish

Benny has to store 36 stuffed animals in boxes.

He puts 9 stuffed animals in each box.

How many boxes does Benny use?

$$36 \div 9 = 4$$

Benny uses 4 boxes.

**Number of Groups Unknown**  
("How many groups?" Division)

$$? \times 6 = 18, \text{ and } 18 \div 6 = ?$$

If 18 plums are to be packed 6 to a bag, then how many bags are needed?

*Measurement example.* You have 18 inches of string, which you will cut into pieces that are 6 inches long. How many pieces of string will you have?

**Math in Focus**

Singapore Math by Marshall Cavendish

# Mathematics I Standards for Mathematical Practice

- Able :
  - to make sense of problems and persevere
  - to reason abstractly and quantitatively
  - to construct viable arguments
  - to model with mathematics
  - to use tools appropriately
  - to attend to precision
  - to looks for and make use of structure
  - to look for regularity

Teachers who foster the practices\_\_\_\_\_.



Teachers who foster the practices\_\_\_\_\_.

Ensure that every child has a mathematical voice.

Teachers who foster the practices\_\_\_\_\_.

Ensure that every child has a mathematical voice.

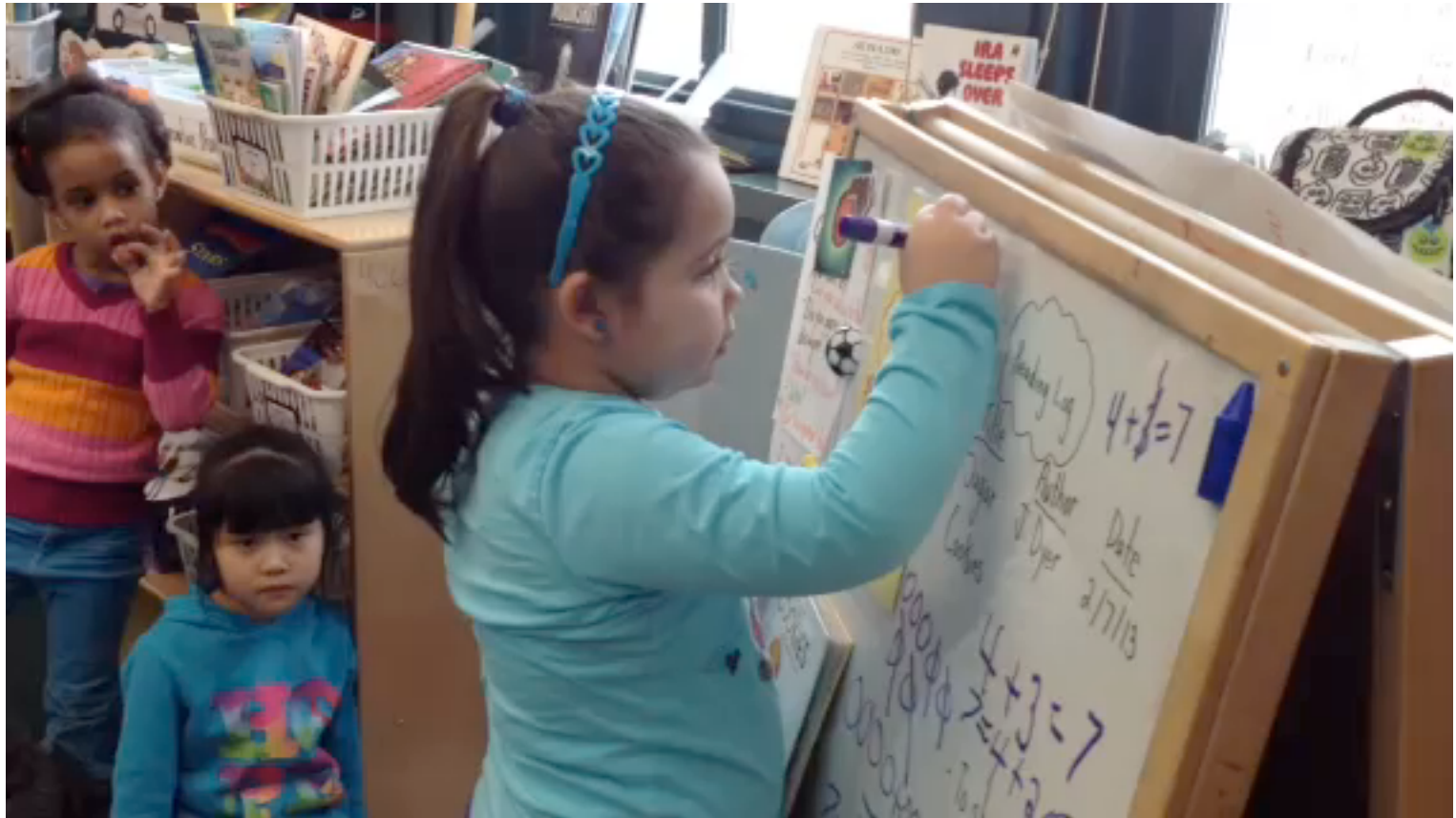
Know when to slow down and what to slow down for.

Teachers who foster the practices\_\_\_\_\_.

Ensure that every child has a mathematical voice.

Know when to slow down and what to slow down for.

Ask the right questions.



# CRESST REPORT      January 2013

## On the Road to Assessing Deeper Learning; The Status of Smarter Balanced and PARCC Assessment Consortia

- Current research estimates that 0% of students in the US were assessed on deeper learning in mathematics through state tests, 16% percent of students were assessed on deeper learning in reading, and 2-3% were assessed on deeper learning in writing.

# Sample NAEP 4th grade sample

## AMUSEMENT PARK

*70 things to do!*

34 rides

plus games

plus shows

19. An amusement park has games, rides, and shows.

The total number of games, rides, and shows is 70.

There are 34 rides.

There are two times as many games as shows.

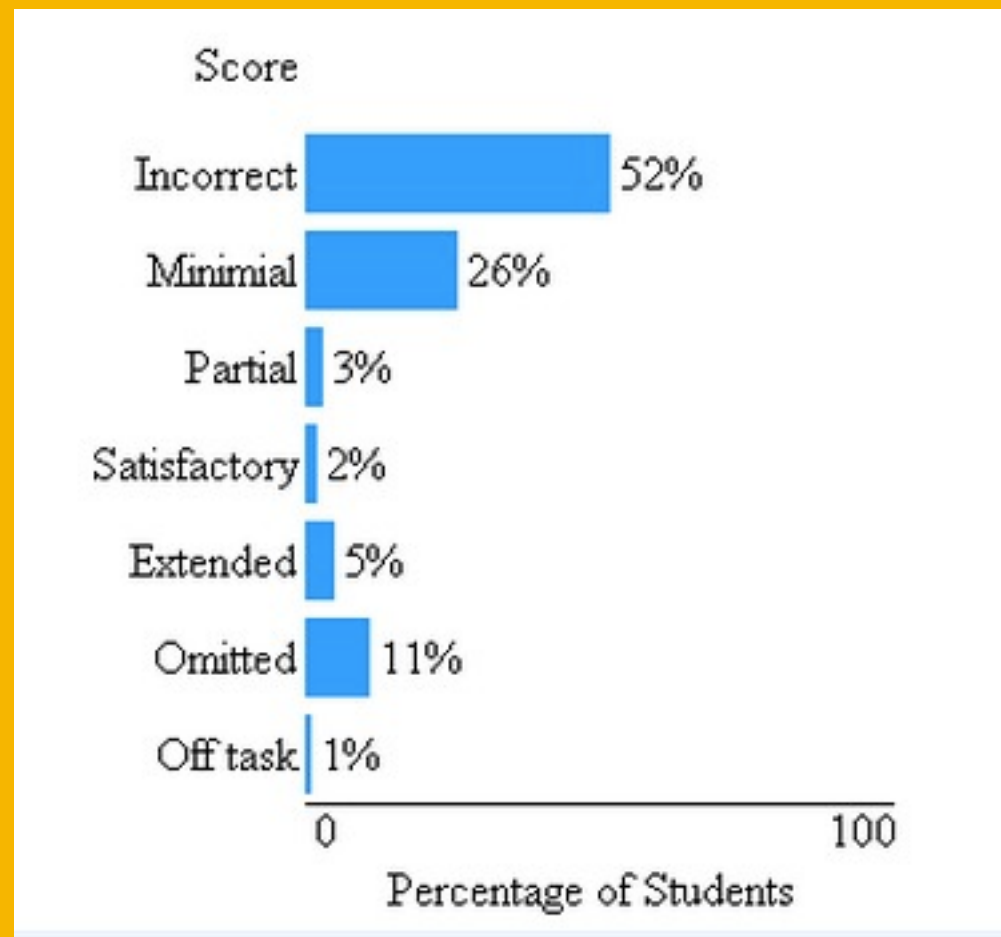
How many games are there? \_\_\_\_\_

How many shows are there? \_\_\_\_\_

Use numbers, words, or drawings to show how you got your answer.

If you need more room for your work, use the space below.

# Sample NAEP4th grade sample



# Sample NAEP 4th grade sample

**Math in Focus**

Singapore Math by Marshall Cavendish



# Sample NAEP 4th grade sample



# Sample NAEP 4th grade sample



# Sample NAEP 4th grade sample

70

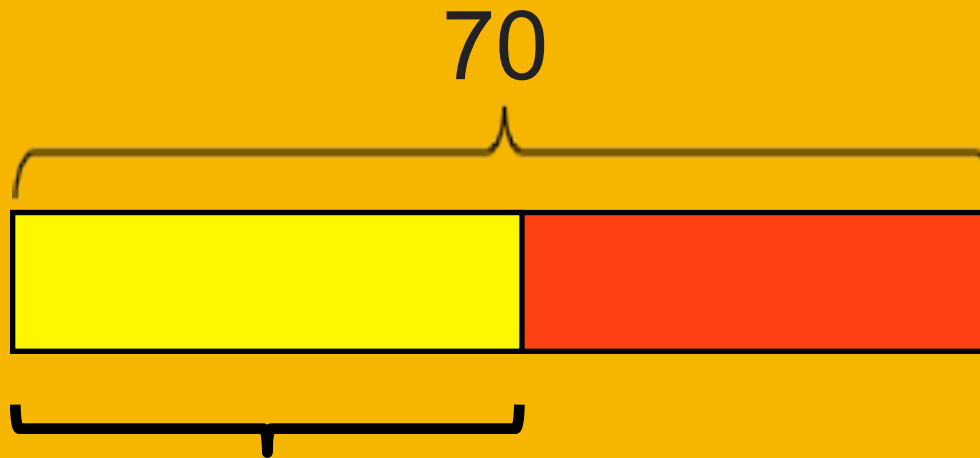


# Sample NAEP 4th grade sample

70



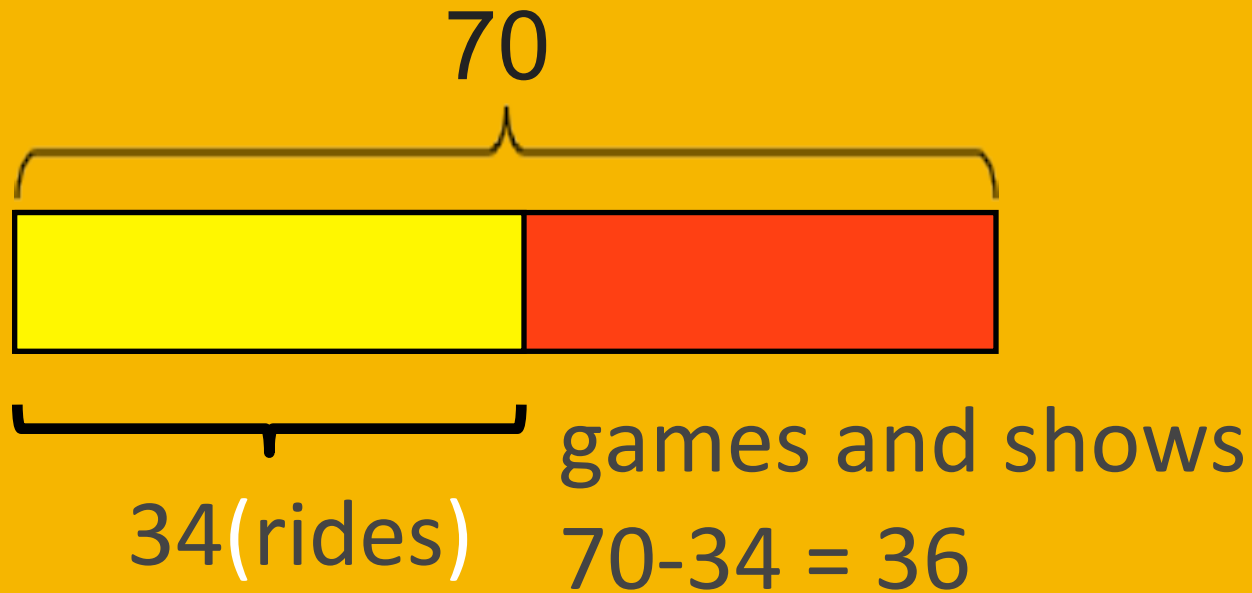
# Sample NAEP 4th grade sample



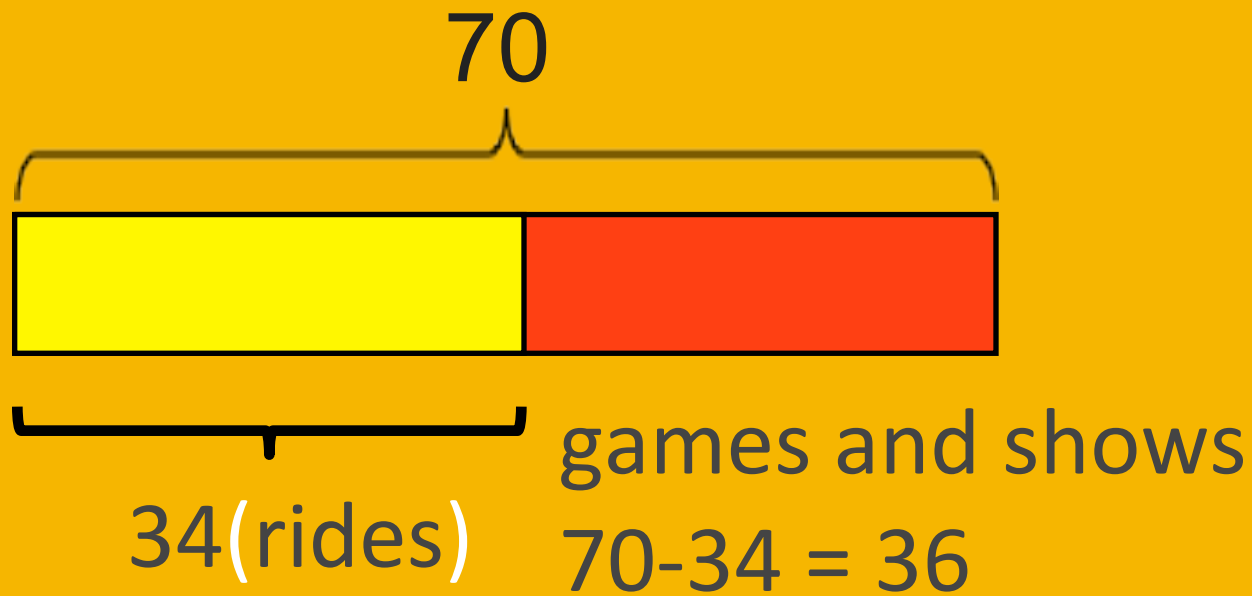
# Sample NAEP 4th grade sample



# Sample NAEP 4th grade sample



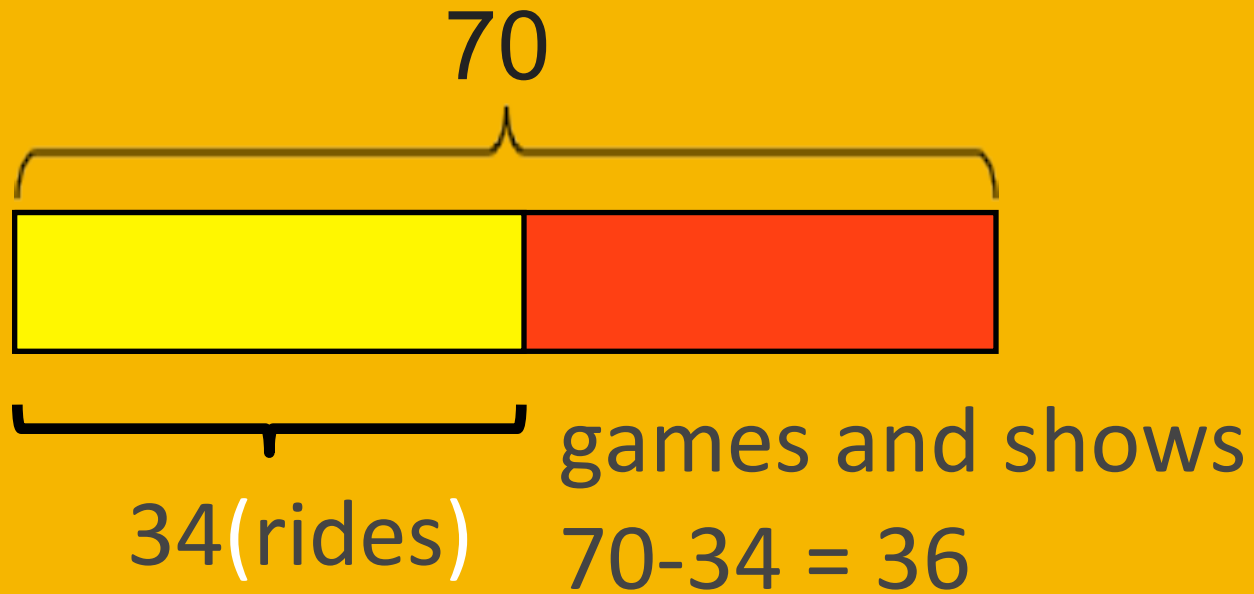
# Sample NAEP 4th grade sample



Shows



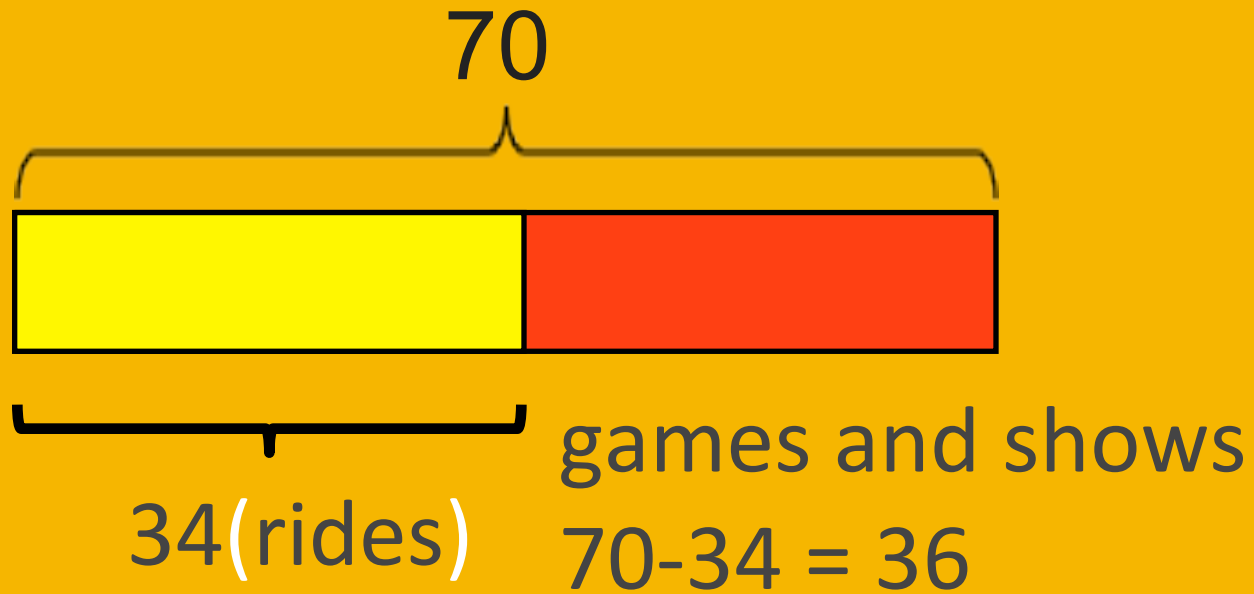
# Sample NAEP 4th grade sample



Shows

Games

# Sample NAEP 4th grade sample

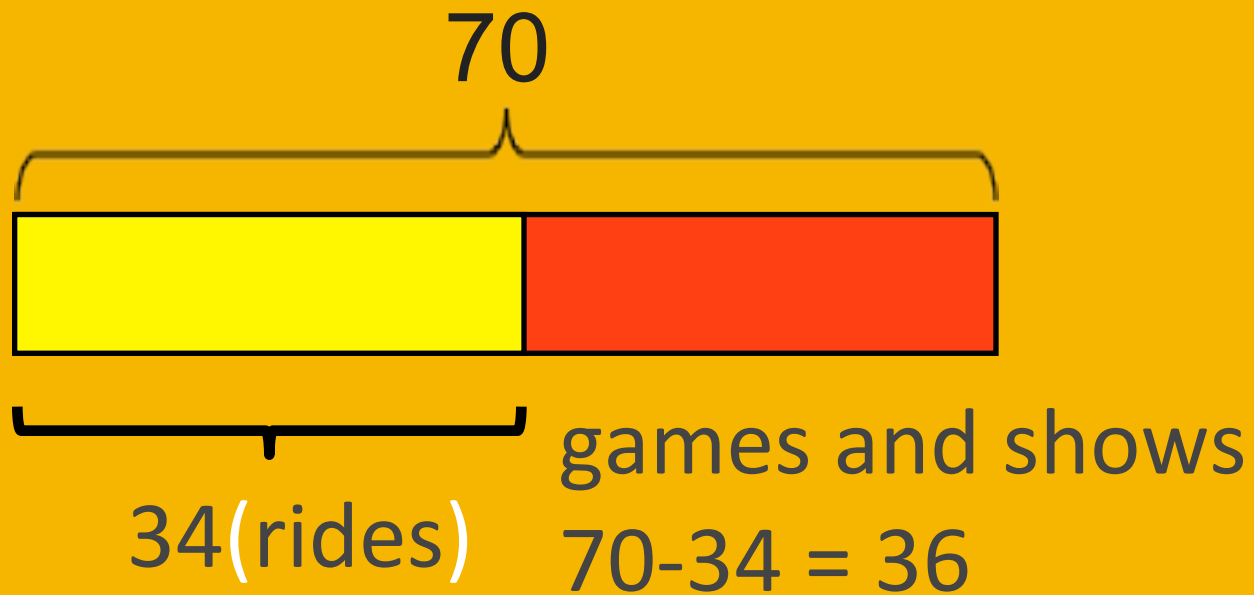


Shows



Games

# Sample NAEP 4th grade sample



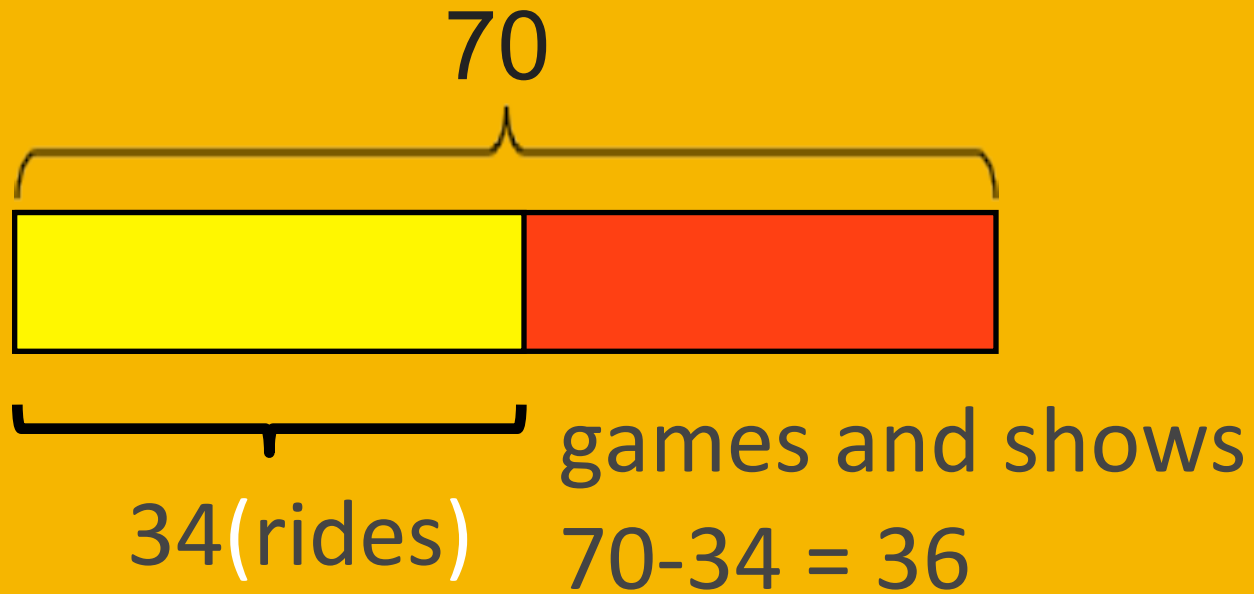
Shows



Games



# Sample NAEP 4th grade sample



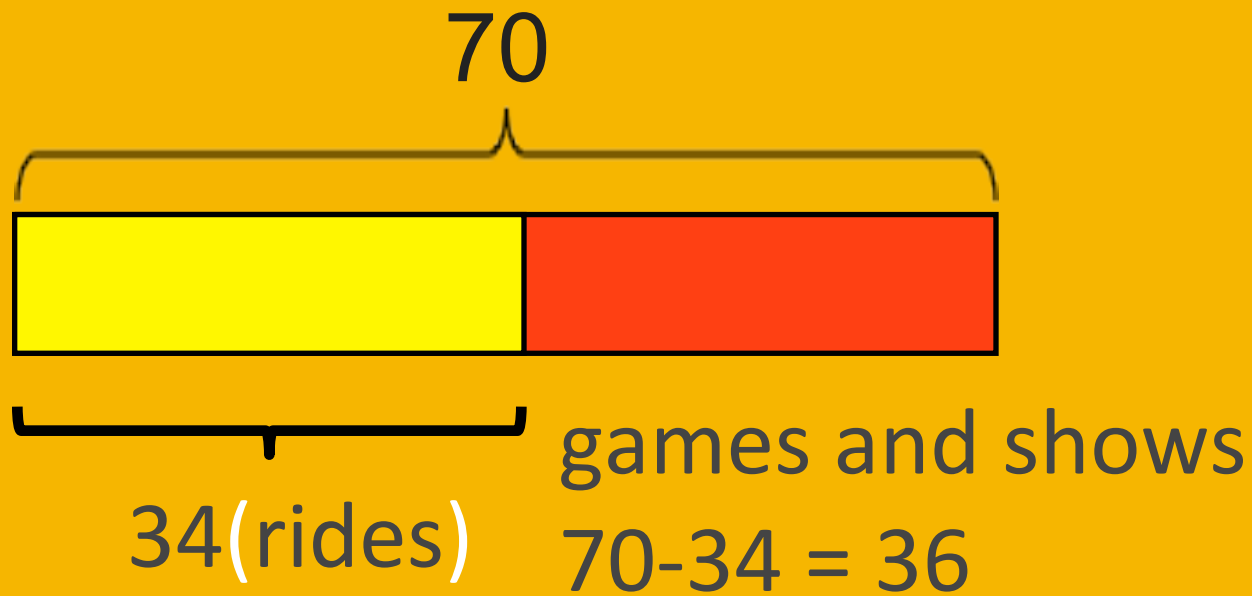
Shows



Games



# Sample NAEP 4th grade sample



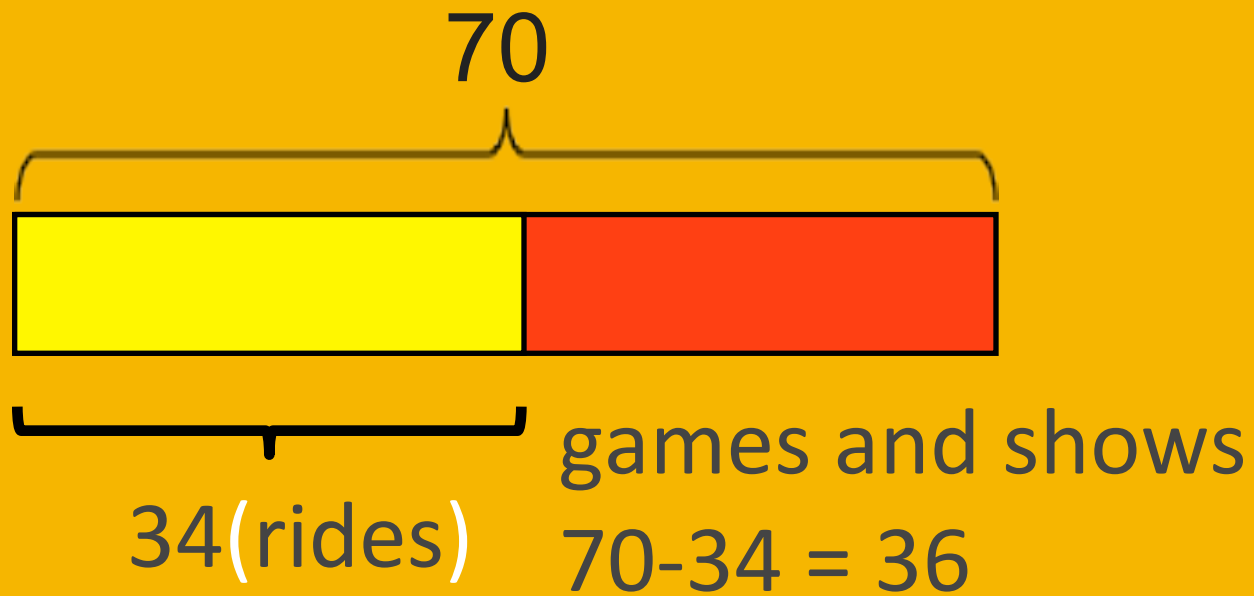
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Games



# Sample NAEP 4th grade sample



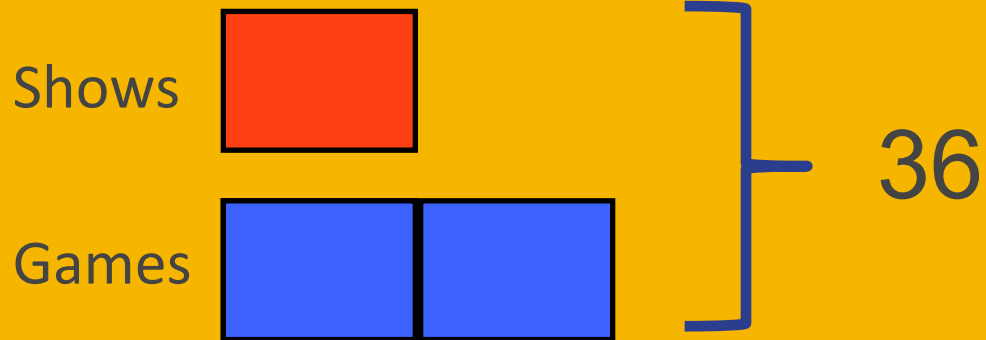
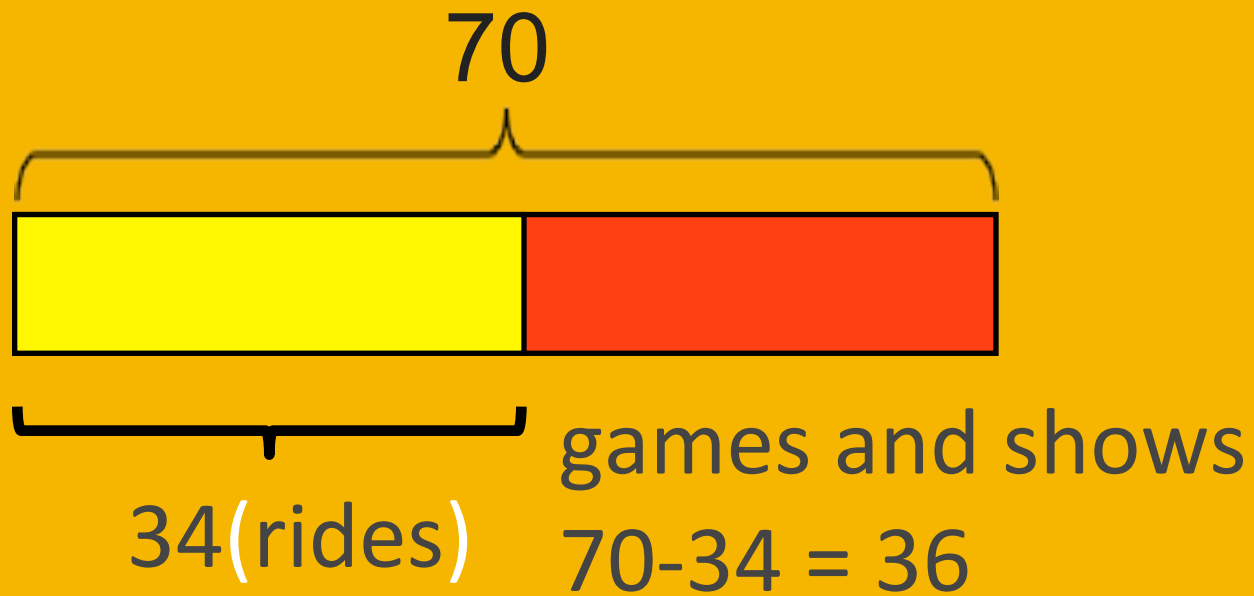
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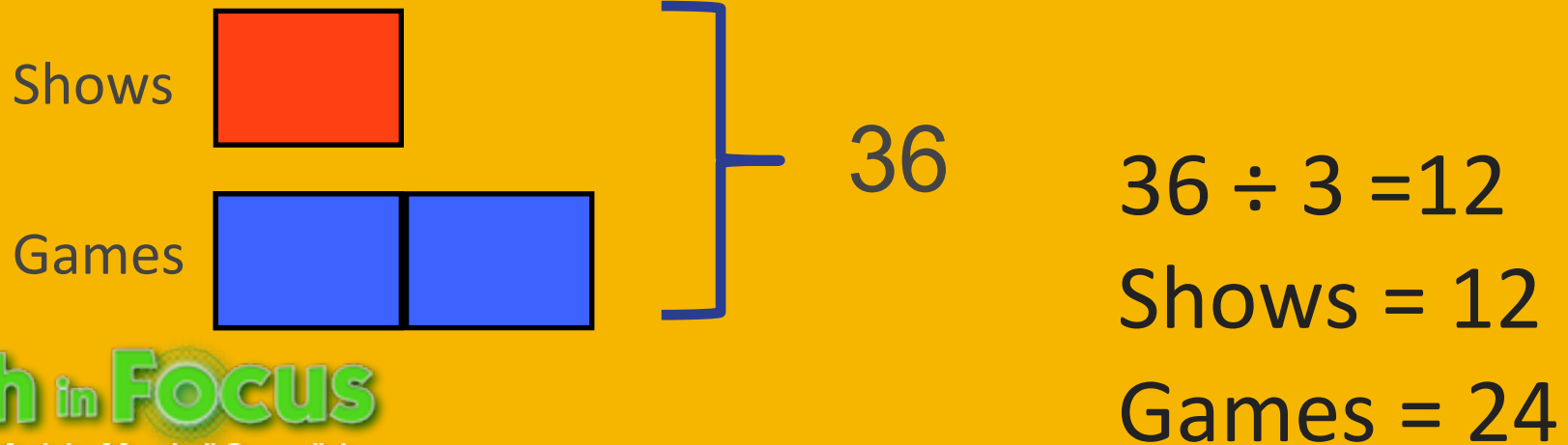
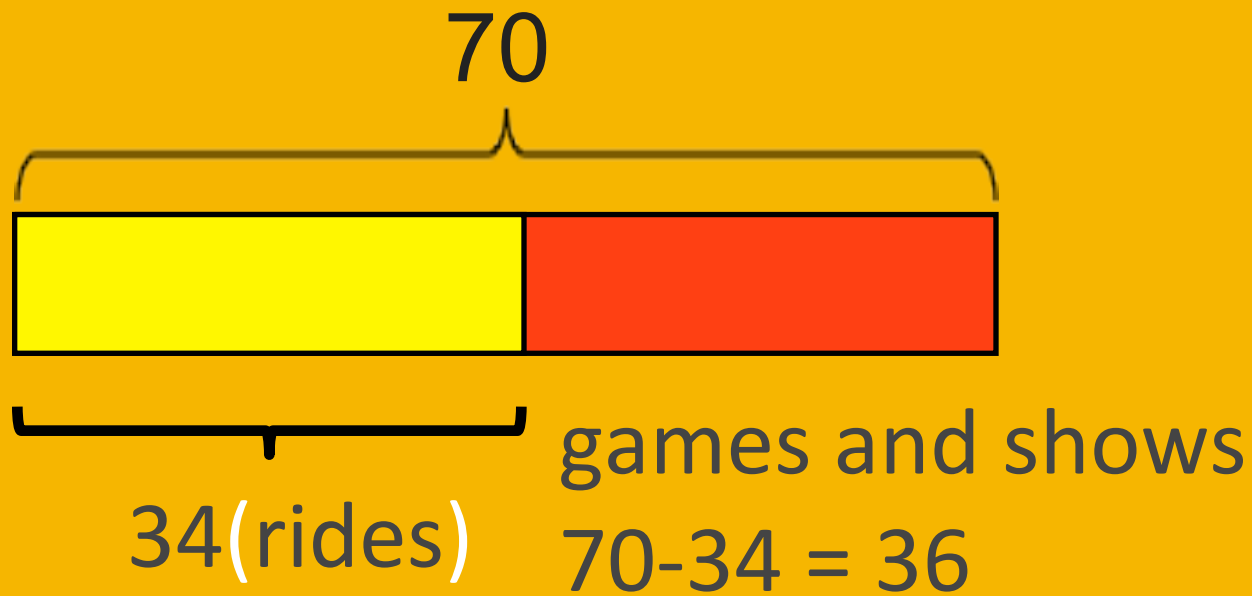
Games



# Sample NAEP 4th grade sample



# Sample NAEP 4th grade sample





**CRESST REPORT      January 2013**

**On the Road to Assessing Deeper Learning; The  
Status of Smarter Balanced and PARCC Assessment**

# **CRESST REPORT      January 2013**

## **On the Road to Assessing Deeper Learning; The Status of Smarter Balanced and PARCC Assessment**

- **End-of-year tests will address each of the mathematical practices.**

# **CRESST REPORT      January 2013**

## **On the Road to Assessing Deeper Learning; The Status of Smarter Balanced and PARCC Assessment**

- End-of-year tests will address each of the mathematical practices.**
- Both consortia plan to assess every student on a full range of DOK or cognitive complexity to encourage schools to provide opportunities for deeper learning for all students. 100% of students in tested grades using the consortia tests (SBAC and PARCC) will be held accountable for deeper learning**

**SMARTER BALANCED – Mathematics** Item Specifications  
 A "Snapshot" of the Cognitive Rigor Matrix (Hess, Carlock, Jones, & Walkup, 2009)

Depth of Thinking Webb + Type of Thinking (Revised Bloom's)	DOK Level 1 Recall & Reproduction	DOK Level 2 Basic Skills & Concepts	DOK Level 3 Strategic Thinking & Reasoning	DOK Level 4 Extended Thinking
<b>Remember</b>	<ul style="list-style-type: none"> <li>Recall conversions, terms, facts</li> </ul>			
<b>Understand</b>	<ul style="list-style-type: none"> <li>Evaluate an expression</li> <li>Locate points on a grid or number on number line</li> <li>Solve a one-step problem</li> <li>Represent math relationships in words, pictures, or symbols</li> </ul>	<ul style="list-style-type: none"> <li>Specify, explain relationships</li> <li>Make basic inferences or logical predictions from data/observations</li> <li>Use models/diagrams to explain concepts</li> <li>Make and explain estimates</li> </ul>	<ul style="list-style-type: none"> <li>Use concepts to solve non-routine problems</li> <li>Use supporting evidence to justify conjectures, generalize, or connect ideas</li> <li>Explain reasoning when more than one response is possible</li> <li>Explain phenomena in terms of concepts</li> </ul>	<ul style="list-style-type: none"> <li>Relate mathematical concepts to other content areas, other domains</li> <li>Develop generalization of the results obtained and strategies used, apply them to new problem situations</li> </ul>
<b>Apply</b>	<ul style="list-style-type: none"> <li>Follow simple procedures</li> <li>Calculate, measure, apply a rule (e.g., rounding)</li> <li>Apply algorithm or formula</li> <li>Solve linear equations</li> <li>Make conversions</li> </ul>	<ul style="list-style-type: none"> <li>Select a procedure and perform it</li> <li>Solve routine problem applying multiple concepts</li> <li>Retrieve information to solve a problem</li> <li>Translate between representations</li> </ul>	<ul style="list-style-type: none"> <li>Design investigation for a specific purpose or research question</li> <li>Use reasoning, planning, and supporting evidence</li> <li>Translate between problem and symbolic notation when not a direct translation</li> </ul>	<ul style="list-style-type: none"> <li>Initiate, design, and conduct a project that specifies a problem, identifies solution paths, solve the problem and reports results.</li> </ul>
<b>Analyze</b>	<ul style="list-style-type: none"> <li>Retrieve information from a table or graph to answer a question</li> <li>Identify a pattern/trend</li> </ul>	<ul style="list-style-type: none"> <li>Categorize data, figures</li> <li>Organize, order data</li> <li>Select appropriate graph and organize &amp; display data</li> <li>Interpret data from a simple graph</li> <li>Extend a pattern</li> </ul>	<ul style="list-style-type: none"> <li>Compare information within or across data sets or texts</li> <li>Analyze and draw conclusions from data, citing evidence</li> <li>Generalize a pattern</li> <li>Interpret data from complex graph</li> </ul>	<ul style="list-style-type: none"> <li>Analyze multiple sources of evidence or data sets</li> </ul>
<b>Evaluate</b>			<ul style="list-style-type: none"> <li>Cite evidence and develop a logical argument</li> <li>Compare/contrast solutions methods</li> <li>Verify reasonableness</li> </ul>	<ul style="list-style-type: none"> <li>Apply understanding in a novel way, provide argument or justification for the new application</li> </ul>
<b>Create</b>	<ul style="list-style-type: none"> <li>Brainstorm ideas, concepts, problems, or perspective related to a topic or concept</li> </ul>	<ul style="list-style-type: none"> <li>Generate conjectures or hypotheses based on observations or prior knowledge and experience</li> </ul>	<ul style="list-style-type: none"> <li>Develop an alternative solution</li> <li>Synthesize information within one data set</li> </ul>	<ul style="list-style-type: none"> <li>Synthesize information across multiple sources or data sets</li> <li>Design a model to inform and solve a practical or abstract situation</li> </ul>

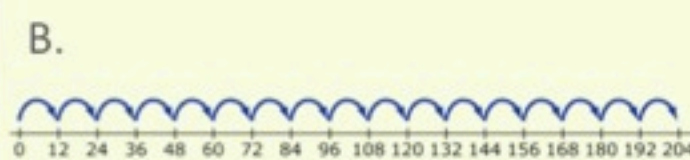
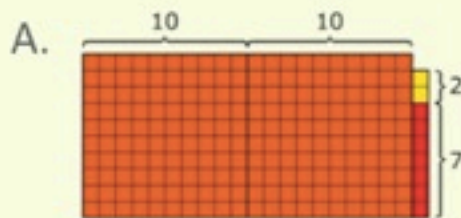
# Preparing for Common Core Assessment

A multiplication problem is shown below.

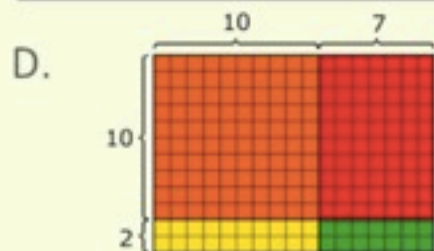
$$17 \times 12$$

STEM

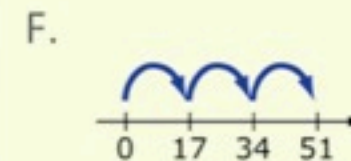
Which model(s) below could represent the solution to this problem?  
Select all that apply.



C.  $(1 \times 1) + (1 \times 7) + (1 \times 2) + (2 \times 7)$



E.  $(17 \times 2) + (17 \times 1)$



# Preparing for Common Core Assessment



Write your answer to the following problem in your answer booklet.

San Francisco  
Giants' stadium:  
41,915 seats

Washington  
Nationals' stadium:  
41,888 seats

San Diego  
Padres' stadium:  
42,445 seats

Compare these statements from two students.

Jeff said, "I get the same number when I round all three numbers of seats in these stadiums."

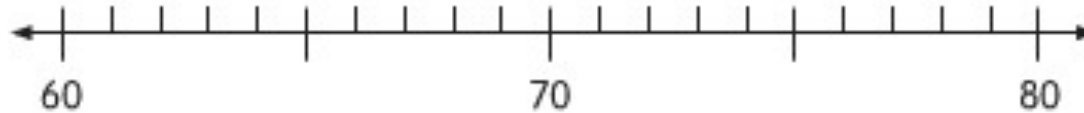
Sara said, "When I round them, I get the same number for two of the stadiums but a *different* number for the other stadium."

Can Jeff and Sara both be correct? Explain how you know.



# Preparing for Common Core Assessment

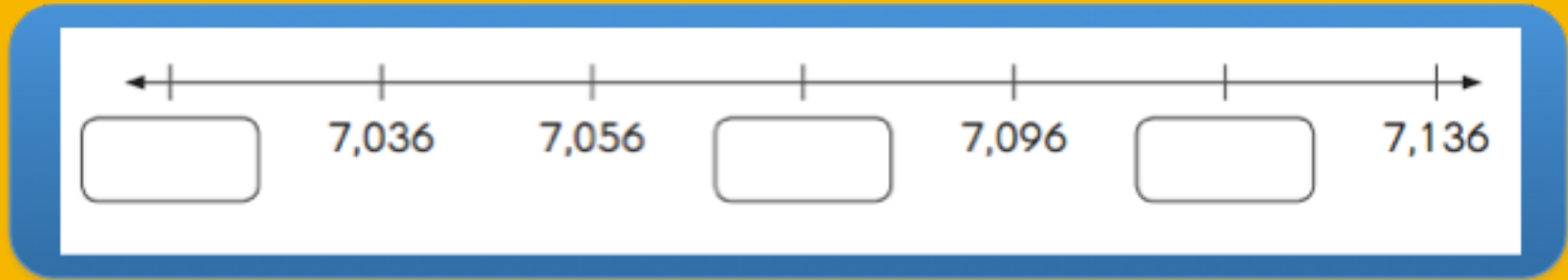
7



A number when rounded to the nearest 10 is 70.

- a Find all the numbers that give 70 when rounded to the nearest ten. Mark each number with a  $\times$  on the number line.
- b Which number is the least?
- c Which number is the greatest?

# Preparing for Common Core Assessment



**Math in Focus**

Singapore Math by Marshall Cavendish



# Preparing for Common Core Assessment

43023

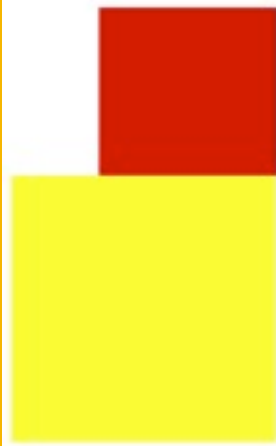


A rectangle is 6 feet long and has a perimeter of  $20\frac{1}{3}$  feet.

What is width of this rectangle? Explain how you solved this problem.

# Preparing for Common Core Assessment

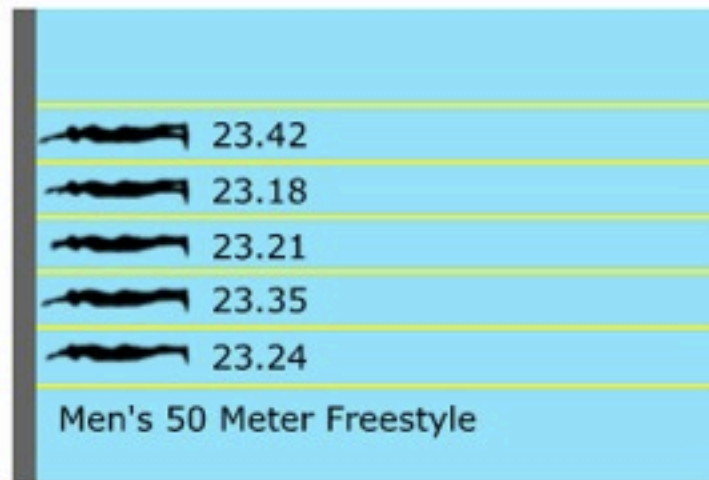
The total area of the two squares is 89, what is the length of the sides of the two squares? What is the perimeter of the figure?



# Preparing for Common Core Assessment

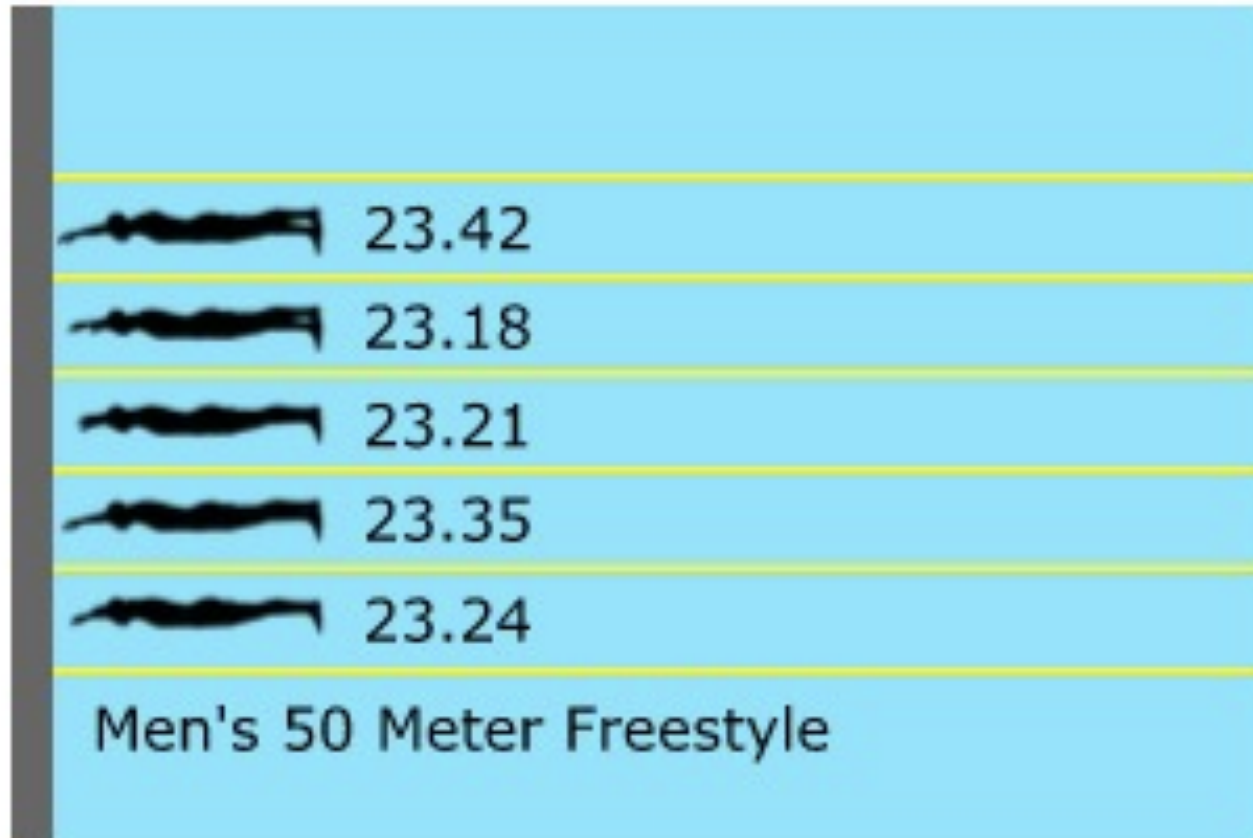
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Five swimmers compete in the 50-meter race. The finish time for each swimmer is shown in the video.



Explain how the results of the race would change if the race used a clock that rounded to the nearest tenth.

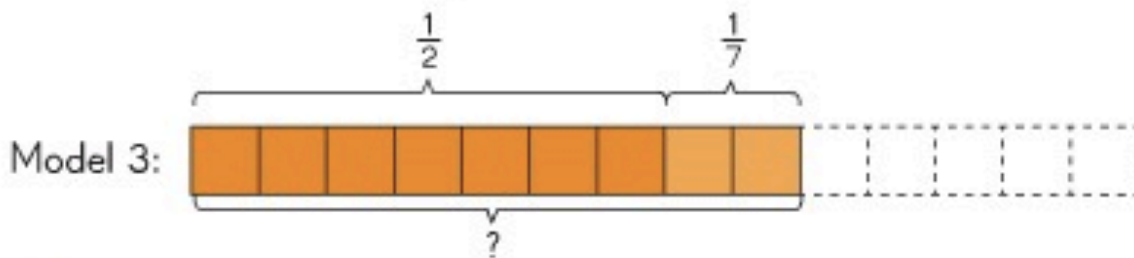
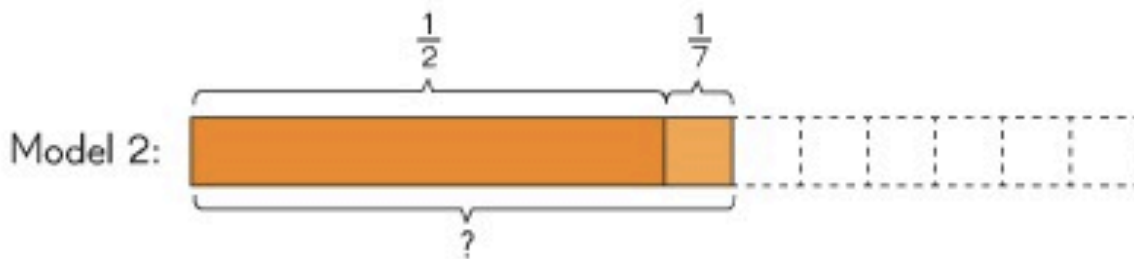
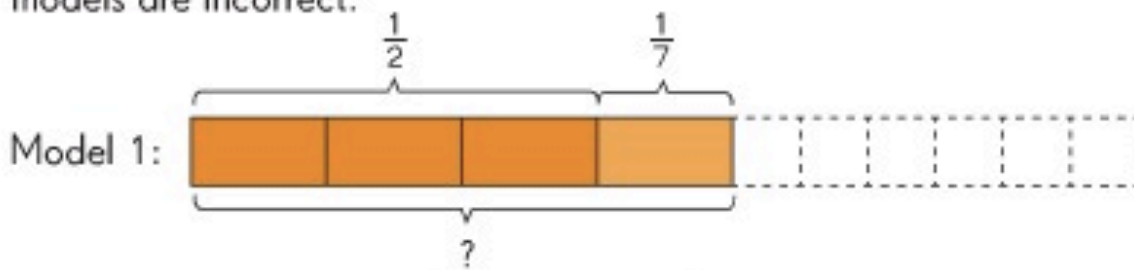
# Preparing for Common Core Assessment



# Preparing for Common Core Assessment

# Math Journal

One of the three models shows the sum of  $\frac{1}{2}$  and  $\frac{1}{7}$ . The other two models are incorrect.



- Identify the correct one of the three.
- Explain why the other two are incorrect.

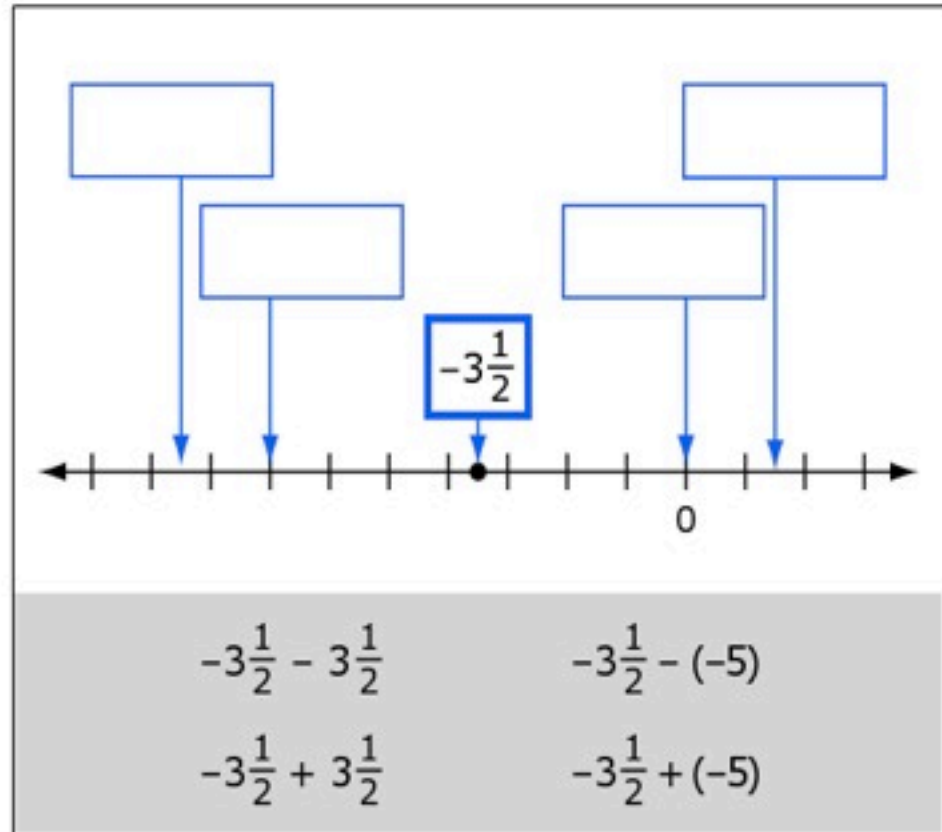
# Preparing for Common Core Assessment

42960



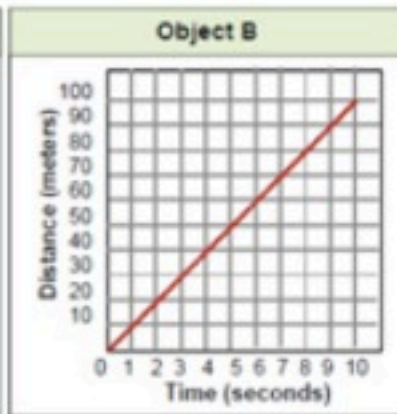
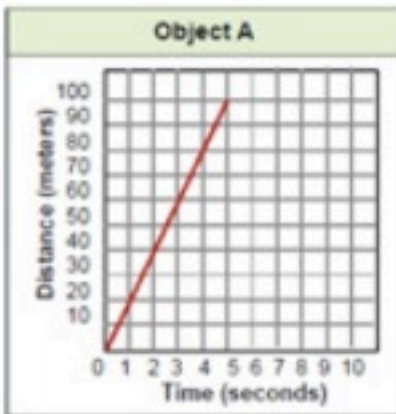
The point on the number line shows the location of  $-3\frac{1}{2}$ .

Move each expression into a box to show its correct location on the number line.



# Preparing for Common Core Assessment

## Grade 7 - Speed



**Object C**

Time (seconds)	Distance (meters)
0	0
3	10
6	20
9	30

Object C moves at constant speed.

**Object D**

Time (seconds)	Distance (meters)
0	0
1.5	10
3	20
4.5	30

Object D moves at constant speed.

The speed of an object is defined as the change in distance divided by the change in time.

Information about objects A, B, C and D are shown. Objects C and D both have constant speed.

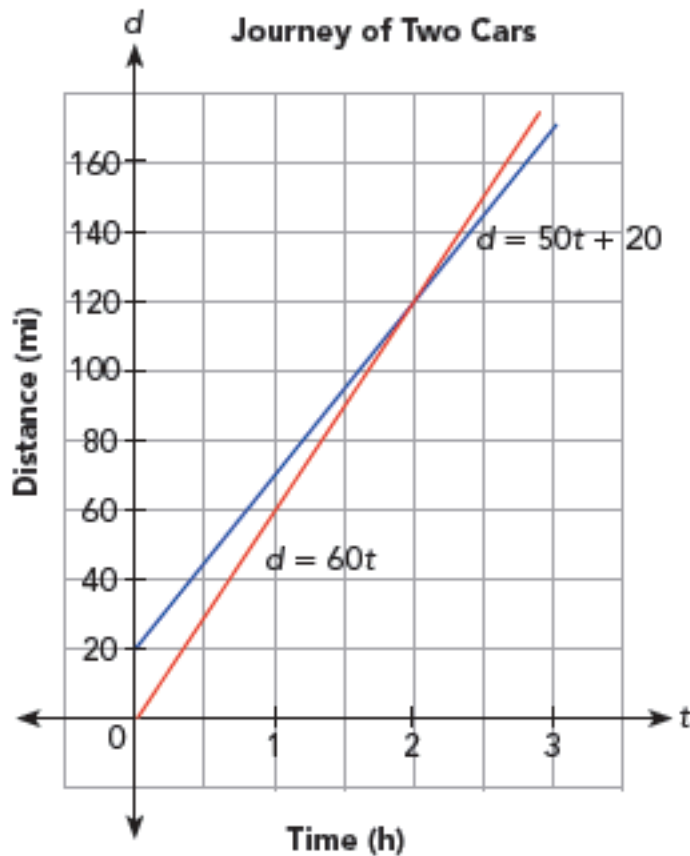
Based on the information given, drag and drop the object names in order from greatest speed to least speed in the table provided.

Object A	Greatest Speed ↓ Least Speed	
Object B		
Object C		
Object D		



# Preparing for Common Core Assessment

Sketch the graphs of the two equations using the slope and y-intercept values.



Because Distance = Rate  $\cdot$  Time, these two equations tell me many things about the journey of the two cars. For example, the speed of each car, whether they start at the same time, and whether they start at the same place.



**Math in Focus**

Singapore Math by Marshall Cavendish

# Preparing for Common Core Assessment

- 13** *Math Journal* The ratio of the number of beads collected by Jane to the number of beads collected by Jill is  $7 : 3$ . Jane gave some beads to Jill. Is it possible for both Jane and Jill to have the same number of beads after Jane gave some beads to Jill? Explain why you think so.

# Preparing for Common Core Assessment

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*Make sense of problems and persevere*

# Preparing for Common Core Assessment

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*Make sense of problems and persevere  
Reason abstractly and quantitatively*

# Preparing for Common Core Assessment

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*Make sense of problems and persevere  
Reason abstractly and quantitatively  
Construct viable arguments*

# Preparing for Common Core Assessment

Not all standards have equal weight

Both PARCC and Smarter Balanced have identified that 75-80% of their assessments will focus on number, number sense, computation, and application to solve problems in the 3-5 assessments, and an emphasis on algebra and number in the 6-8 curriculum.

Book A of MIF contains the majority of the tested topics and will require somewhat more time.


# Where do we find the practices?

- Guided practice
- Let's Practice
- Let's Explore
- Hands On
- Put On your Thinking Cap
- Math Journal

**CRITICAL THINKING SKILLS**  
**Put On Your Thinking Cap!**

**PROBLEM SOLVING**

1 The model shows  $\frac{3}{4}$  of a fraction strip shaded. How many of the shaded parts must be erased so that the remaining shaded parts are  $\frac{1}{4}$  of the strip?



**READING AND WRITING MATH**  
**Math Journal**

List the steps to arrange the numbers in order from least to greatest.

Example

1,984 2,084 1,884

1 I compare the thousands.

**Hands-On Activity**

**WORK IN PAIRS**

Players: 2  
Materials: • worksheet

1 Player 1 thinks of a 4-digit number with 1, 2, 3, and 4. Use each digit only once.

2 Player 2 writes his or her first guess in the first row of the worksheet.

Thousands	Hundreds	Tens	Ones
1	2	4	3

3 Player 1 gives some clues. For example, if Player 1's number is 2,314 and Player 2's guess is 1,243, Player 1 says:

- My thousands is greater than yours.
- My hundreds is greater than yours.
- My tens is less than yours.
- My ones is greater than yours.

- <http://www.achievethecore.org/>  
*This site is assembled by Student Achievement Partners to provide free, high-quality resources to educators now doing the hard work of implementing these higher standards.*
- <http://www.illustrativemathematics.org/>  
*“Illustrative Mathematics provides guidance to states, assessment consortia, testing companies, and curriculum developers by illustrating the range and types of mathematical work that students experience in a faithful implementation of the Common Core State Standards, and by publishing other tools that support implementation of the standards.”*
- <http://pta.org/parents/content.cfm?ItemNumber=2583&RDtoken=51120&userID>  
*The PTA’s Parents’ Guide to Student Success (in English and Spanish) was developed in response to the Common Core State Standards. The Guide includes key items that children should be learning and activities that parents can do at home to support their child's learning.*
- <http://commoncoretools.me/>  
*News about tools that are being developed to support implementation of the Common Core State Standards for Mathematics.*
- <http://www.corestandards.org/>  
*The website that hosts the complete CCSS documents as well as a collection of resources.*
- <http://www.parcconline.org/>  
*The website of the PARCC (Partnership for Assessment of Readiness for College and Career) Consortium.*



<http://www.pta.org/4thGradeFeb4.pdf>



**National**  
**PTA**  
*everychild.onevoice.®*

## PARENTS' GUIDE TO Student Success

### 4TH GRADE

This guide provides an overview of what your child will learn by the end of 4th grade in mathematics and English language arts/literacy. It focuses on the key skills your child will learn in these subjects, which will build a strong foundation for success in the other subjects he or she studies throughout the school year. This guide is based on the new Common Core State Standards, which have been adopted by more than 40 states. These K-12 standards are informed by the highest state standards from across the country. If your child is meeting the expectations outlined in these standards, he or she will be well prepared for 5th grade.

#### WHY ARE ACADEMIC STANDARDS IMPORTANT?

Academic standards are important because they help ensure that all students, no matter where they live, are prepared for success in college and

#### HOW CAN I HELP MY CHILD?

You should use this guide to help build a relationship with your child's teacher. You can do this by talking to his or her teacher regularly about how your child is



# Common Core, Assessments and MIF





# Common Core, Assessments and MIF

Answer Getting?





# Common Core, Assessments and MIF





# Common Core, Assessments and MIF

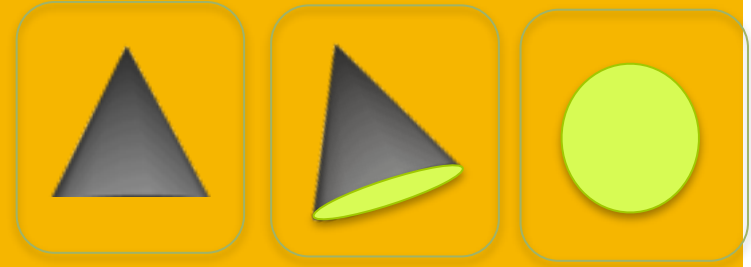
or  
Learning Mathematics





# Common Core, Assessments and MIF





# Common Core, Assessments and MIF

Fast Thinking or ...

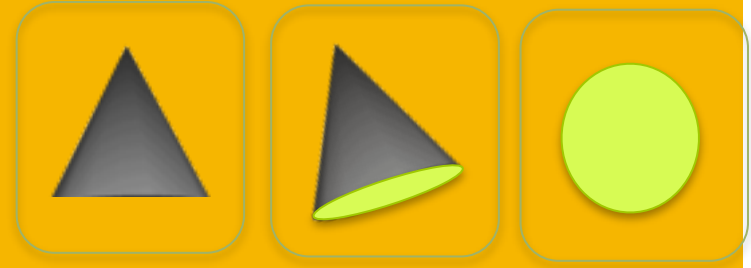




# Common Core, Assessments and MIF







# Common Core, Assessments and MIF

slow thinking





# Common Core, Assessments and MIF



Andy Clark 7/17/13

# Common Core, National Assessments and *Math* *In Focus*

Meeting Common Core and World Class  
standards

